Predicting the Atlantic Meridional Overturning Circulation Using Nonlinear System Identification Methods and the NARMAX Model

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Abstract

The Atlantic Meridional Overturning Circulation (AMOC) plays an important role in the coupled ocean-climate system and in global climate change. The analysis of its own behaviour and the understanding its links to other climate dynamics is of paramount importance today as we encounter an increasing pressure to adapt to climate change. Due to the enormous complexity, it is almost impossible to establish accurate models, purely based on first-principle modelling approaches, that can perfectly represent the relationships between the AMOC and other dynamic climate parameters. Data-based or data-driven modelling methods, can therefore provide an attractive alternative solution. Systematic regular and continuous measurement of the AMOC time series began in April 2004. The main objective of the paper is to use the monthly data of the AMOC measured during April 2004-Febuary 2017, together with the North Atlantic Oscillation (NAO) index, and density anomalies of the Gulf of Mexico, Labrador Sea and Norwegian Sea, measured during the same period, to investigate and understand the quantitative relationship between the AMOC and four drivers (NAO and the three density anomaly variables). In doing so, nonlinear system identification methods and the Nonlinear AutoRegressive Moving Average with Exogenous input (NARMAX) method are employed to develop a quantitative model that relates the AMOC to the four drivers. Experimental results show that the derived nonlinear model skillfully captures and represents the dynamics of the AMOC based on the other four variables. One of the findings from this study is that the use of autoregressive variables can help improve the prediction of the AMOC.

Keywords: atlantic meridional overturning circulation, north atlantic oscillation, nonlinear system identification methods, narmax model

Introduction

The Atlantic Meridional Overturning Circulation (AMOC) is a large complex system of ocean currents that circulate water within the Atlantic Ocean, typically bringing warm water from the tropics northwards into the North Atlantic. The AMOC plays a key role in understanding the past, present and future atmospheric climate changes [1]. Due to technical limitations and other factors in the past, regular and continuous measurement of the AMOC only became available in 2004 with the deployment of the RAPID array and related time series array instruments along 26ºN [2]. The resulting availability of the AMOC data has given rise in recent years to many empirical and theoretical studies of the AMOC, that focus on revealing the sources or causes of variation of this key ocean circulation system. A generally agreed finding is that density anomalies along the western boundary current [3], and particularly in the Labrador Sea [4], make leading contributions to the variation of the AMOC, as evidenced by the southward propagation of boundary waves instead of water masses [4][5].

In recent years, there has been increasing interest in predicting the AMOC (see, e.g., [3][6][7]), as skillful predictions of the AMOC are highly valuable and crucially important for better understanding the past behaviour of climate, as well as better monitoring the future behaviour [8]-[10]. A skillful prediction of the AMOC may also probably help prevent or circumvent major climatic hazards in future.

In [7], a complex nonlinear system identification and modelling approach was employed to build predictive models based on data of the period of April 2004 – March 2014. The resulting models were used to hindcast the variability of the AMOC between 1980 and 2004. A total of four drivers were considered for model construction, namely, the North Atlantic Oscillation (NAO) index, density anomalies of the Gulf of Mexico (GM), Labrador Sea (LS) and Norwegian Sea (NS). Two new variables were derived from the three density anomalies: the first one is defined as the mean of the density variables, U=(GM+LS+NS)/3; the second is defined as the atmosphere and the meridional density difference between surface and deep waters, V=(LS+NS)/2-GM. Note that the AMOC was not well measured (not measured continuously) before 2004 so the models developed in [7] did not use any autoregressive variables of the AMOC, such as AMOC(*t*-1) and AMOC(*t*-2) (the value at time instants *t*-1, *t*-2), when predicting the value of the AMOC at the present time instant *t*. One of the main findings from [7] was that the NAO index plays an apparently significant role in explaining the variation of the AMOC strength, which was further confirmed by a recent study [11].

Motivated by the aforementioned results and findings, and in particular the desire to improve predictive capability for the AMOC, this paper attempts to answer the following questions: 1) How is the AMOC variability quantitatively driven by the NAO index, and the density anomalies of the Gulf of Mexico, Labrador Sea and Norwegian Sea? 2) Can the use of autoregressive variables help improve the prediction of the AMOC and meanwhile maintain the good interpretability of the models? An empirical study is performed based on the monthly data of the AMOC measured at 26°N in the period of April 2004 – February 2017, and data of the three drivers: NAO, U and V. We employ nonlinear complex system identification and modelling methods, including the Nonlinear AutoRegressive Moving Average with Exogenous input (NARMAX) model, to investigate and reveal the relationship between the system output (the AMOC) and the inputs (the three drivers). The implementation of the NARMAX models observes the following principle [12]-[14]: to build white-box models for complex black-box systems. Resulting models are usually transparent, interpretable, parsimonious and sparse (TIPS) and relatively simple compared to other machine learning methods whose models are usually opaque and therefore lack interpretability. Such complex system identification approaches and TIPS models to be used have been recently successfully applied to environmental and weather processes, including the analysis and modelling of iceberg discharge from the Greenland Ice Sheet [15]-[17], the response of cod fish population to environmental changes [18], and the modelling for statistical forecasting of winter North Atlantic atmospheric variability [19].

The main contributions of the paper are as follows:

1) Empirical experiments, with and without using lagged autoregressive variables, such as AMOC(*t*-1), as model inputs are carried out and nonlinear models are developed accordingly.
2) One of the findings from the models is that if autoregressive variables are allowed to enter into the model, then the model first term is AMOC(*t*-1), the one-month lagged version of the AMOC, meaning that the AMOC time series is highly linearly dependent on or correlated to its immediately previous state. It is also noted that the value of AMOC, at the present time instant *t*, is significantly nonlinearly dependent on its other two previous values AMOC(*t*-3) and AMOC(*t*-4) but not on AMOC(*t*-2).
3) Another finding, revealed by the overall prediction skills of the models, measured by root mean square error, mean absolute error and correlation coefficient between observations and predictions, is that the use of autoregressive variables can help improve the prediction of the AMOC.

Data

Following [7], this study uses the monthly observations of the AMOC (unit: Sverdrups - Sv, or 10^6 m^3 s^{-1}), measured during the period of April 2004 – February 2017, involving a total of 155 values which are shown in Figure 1. The potentially influential variables are chosen to be NAO, density anomalies (unit: kg m^{-3}) for the Gulf of Mexico (GM), Labrador Sea (LS) and Norwegian Sea (NS), measured during the period of April 2004 – February 2017. The two new derived variables U (kg m^{-3}) and V (kg m^{-3}) are defined as follows:

$$U = \frac{1}{3}(GM + LS + NS) \tag{1}$$

$$V = \frac{1}{2}(LS + NS) - GM \tag{2}$$

In this study, the AMOC is treated to be the system output (response) variable, and NAO, U and V are treated to be the input (independent) variables; these four variables will be used to build nonlinear predictive models for the AMOC. Detailed descriptions of these variables and associated dataset can be found in [7].

The data are split into three parts. The first part contains a total of 108 monthly data points, measured during the period between April 2004 and March 2013; these samples are used for model training. The second part, containing 12 samples measured in the period of April 2013 – March 2014, is used for model validation. The third part, containing 35 samples measured in the period of April 2014 – February 2017, is used for model testing.

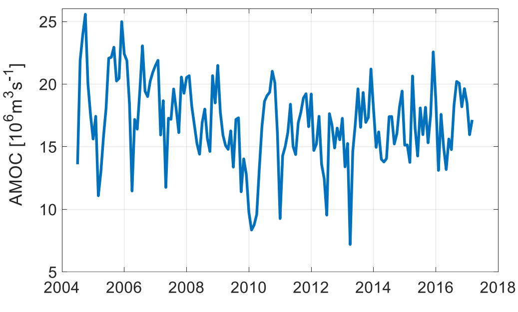

Fig. 1. Monthly measurements of the AMOC during the period of April 2004 – February 2017. The AMOC units are Sverdrups (Sv) or $10^6 m^3 s^{-1}$.

The raw values of the seven involved variables, AMOC, NAO, GM, LS, NS, U and V, are shown in Figures 1 and 2, where it can be noted that the amplitude of GM, LS, NS and U are much large than that of AMOC, NAO, and V. To reduce and balance the large difference in amplitude between these variables, the four variables, AMOC, NAO, U and V, which are directly used for model construction, are pre-processed by removing their mean values as follows:

$$x = x_{raw} - \overline{x}_{train,mean} \tag{3}$$

where x can be any of the four variables. Note that we use the symbol $\overline{x}_{train,mean}$ to highlight that the mean value must be calculated from the training data, and it must be used as an estimate of the mean value of the corresponding variable when needed during the validation, test and prediction processes.

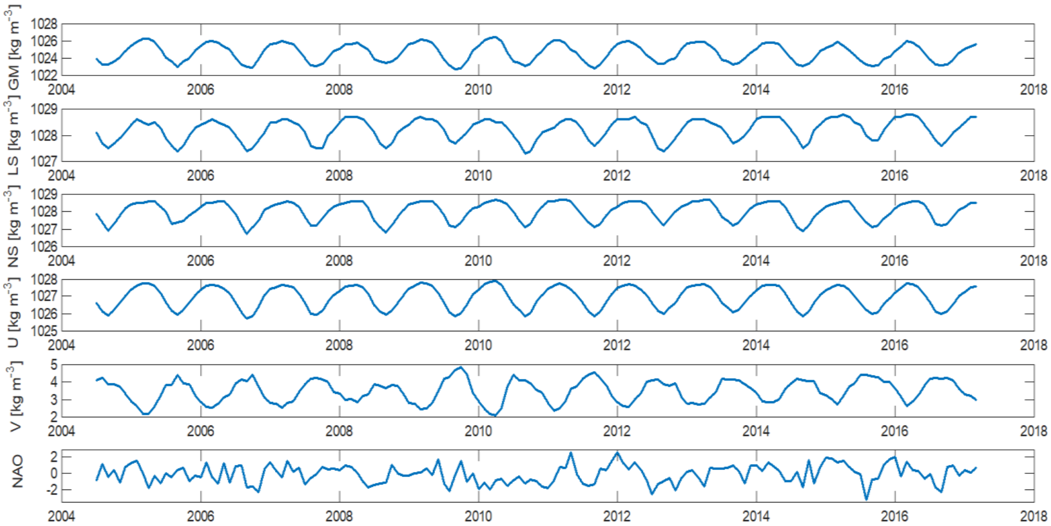

Fig. 2. Monthly observed values between April 2004 and February 2017. From the top panel to the bottom one: density anomalies at Gulf of Mexico (GM), Labrador Sea (LS) and Norwegian Sea (NS); the two derived variable U and V; and standardized NAO index.

The mean-removed monthly values of AMOC, NAO, U and V, for the period of April 2004 – February 2017, are shown in Figure 3. The pre-processed values are directly used to implement the model building procedure.

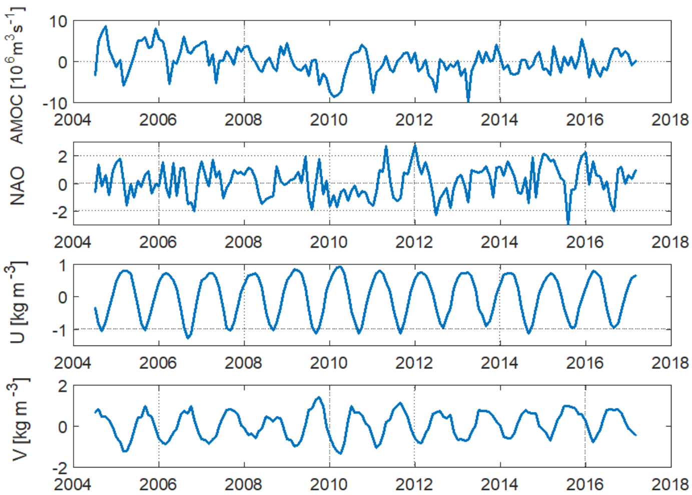

Fig. 3. Mean removed monthly values of AMOC, NAO, U and V, measured between April 2004 and February 2017.

Methods

Findings in the existing literature outlined in section 1 support the following hypothesis: The AMOC variability is quantitatively driven by the NAO index, and the density anomalies of the Gulf of Mexico, Labrador Sea and Norwegian Sea. However, the quantitative relationship from the drivers to the AMOC response is unknown, and the accurate first-principal models may never be known due the enormous complexity of the systems. Fortunately, nowadays regularly measured values of variables of interest are readily available, making data-driven modelling be an indispensable alternative approach to finding a solution.

This study uses the NARMAX (Nonlinear AutoRegressive Moving Average with Exogenous input) method [12], which was initially developed for solving complex control and systems engineering problems. The NARMAX method provides a transparent, interpretable, parsimonious and sparse (TIPS) machine learning approach. The method complies with the following procedures:

1) Data preparation. A data collection, X_t, for a number of input variables of interest, and another data collection, Y_t, for the desired output (response) variable, should be well arranged. It is assumed that the system behavior at the present time instant t is dependent on the previous input and output state values.
2) A model type choice. NARMAX models can be implemented using different basis functions, such as polynomial functions, radial basis functions and wavelets. Polynomial functions, due to their attractive features, are commonly used for NARMAX model construction.

3) Determination of model hyper-parameters. The determination of model hyper-parameters, such as the time delays between the input and output signals, the nonlinear degree of the models (i.e. the allowable highest-order model terms), and the maximum lags (which determine how many lagged input and output variables are used for model construction), is important and may be challenging in many applications.
4) Model construction. The NARMAX method uses several different statistical and sparse learning algorithms to find a group of best models from a huge number of candidate models through the designed model validation and verification processes.

For a system with three inputs and one output, as for the AMOC modelling and prediction problem here, the NARMAX model can be written as

$$\begin{aligned} y(k) = f(&y(k-1),...,y(k-p),u_1(k-d),...,u_1(k-q),u_2(k-d),...,u_2(k-q), \\ &u_3(k-d),...,u_3(k-q),e(k-1),...,e(k-r)) + e(k) \end{aligned} \tag{4}$$

where $u_1(k)$, $u_2(k)$ and $u_3(k)$ are system inputs, $y(k)$ is system output, and $e(k)$ is noise; p, q and r are the associated maximum time lags; d is the time delay, which for many processes can be set as $d = 0$ or $d = 1$; $f(\cdot)$ is an unknown function that needs to be built from available training data. The noise signal $e(k)$ cannot be measured in real applications, but in practice it can be approximated using the model prediction error $\varepsilon(k) = y(k) - \hat{y}(k)$, where $\hat{y}(k)$ is the model prediction at time instant k.

The nonlinear degree of the NARMAX model is defined as the highest order of all model terms. For example, the nonlinear degree of the model $y(k) = a_0 + a_1y(k\text{-}1) + a_2u(k\text{-}1)$ is 1, whereas the nonlinear degree of the model $y(k) = a_0 + a_1y(k\text{-}1) + a_2y(k\text{-}1)[u(k\text{-}2)]^2$ is 3.

In this study, the time delay d is chosen to be '1', which is different from that used in [7] where d =0. This is because the model to be developed in this study is primarily used to make one-month ahead prediction of the AMOC; in doing so the estimation of the value at the present time instant k, AMOC(k), should not use the values of the inputs at the same instant k. Following [7], the maximum lags p, q and r are set to be 8, 4, and 8, which were suggested by simulation experiments guided by the methods proposed in [13].

The nonlinear degree of models is set to be 3; this usually (but not always) enables the finding of better models than a smaller nonlinear degree (e.g. 1 or 2), because a larger nonlinear degree means a larger model library. The forward regression orthogonal least squares (FROLS) algorithm [12] is used to choose the best model terms, and cross-validation techniques including an adjustable prediction error sum of squares (APRESS), also known as the adjustable generalised cross-validation (AGCV) [20], is used to control the model complexity.

A final predictive model established for the AMOC can be represented as

$$\begin{aligned} y(k) = f^{[SysDy]}(&AMOC(k-1),\ ...\ ,AMOC(k-4),\ U(k-1),\ ...\ ,U(k-8),\ V(k-1),\ ...\ ,V(k-8), \\ &NAO(k-1),\ ...\ ,NAO(k-8)) \end{aligned} \tag{5}$$

where $f^{[SysDy]}(\cdot)$ is a NARX (Nonlinear AutoRegressive with Exogenous input) model for the process dynamics only, which is the deterministic part of the NARMAX model (4). The moving average submodel, which is only used for noise estimation and model refinement during the model building process, is removed for later analysis and prediction purposes.

Results

As mentioned earlier, the values of all the variables used for model building have had their mean values removed. The 155 monthly data values are split into three parts: 108 samples (April 2004 and March 2013) for model training; 12 samples (April 2013 – March 2014) for validation; and 35 samples (April 2014 – February 2017) for model test.

Using the knowledge obtained in our previous study (e.g. [7]) and further simulation experiments carried out, the main model hyper-parameters involved in the NARMAX model (4) are as follows. The maximum lag p in the response variable AMOC was set to 4; the maximum q in the input variables NAO, U and V were set to 8; the maximum lag r in moving average variable (model residual) was set to be 8; the time delay d was set to 1; and the nonlinear degree of the model was set to 3.

Using the methods described in the previous section, a nonlinear model consisting a total of 11 model terms was obtained, which is shown in Table 1. Note that the model shown in Table 1 should read:

$$AMOC(k) = 0.6022 \times AMOC(k\text{-}1) + 0.6916 \times NAO(k\text{-}1) \times NAO(k\text{-}1) \times U(k\text{-}7) + \dots \tag{6}$$

and AMOC strength predicted by the model is:

$$AMOC^{[O]}(k) = AMOC(k) + 16.98 \tag{7}$$

In (7), the mean-removed value is mapped back to its original or actual value by adding the mean back.

From Table 1, we have the following observations:

1) The AMOC is driven by the NAO index, variable U (the mean of the density), and V (the difference between the density variables), collectively and nonlinearly. There are some complex interactions among these three drivers which potentially affect the variability of the AMOC.
2) The model first term is AMOC(t-1), meaning that the AMOC time series is highly linearly dependent on or correlated to its immediately previous state. It is also noted that the value of AMOC, at the present time instant t, is significantly nonlinearly dependent on its other previous values such as AMOC(t-3) and AMOC(t-4), but the lagged variable AMOC(t-2) does not appear in the model.

3) The second model term, NAO(k-1) × NAO(k-1) × U(k-7), suggests that the two quantities, the strength of NAO observed one month ago and the value of U (mean of the density) measured seven months ago, coupling together may affect the present behavior of the AMOC.

A graphical illustration of the comparison between the model predicted values and the corresponding observations is shown in Figure 4. Three metrics, namely, root mean square error (RMSE), mean absolute error (MAE), and correlation coefficient (CC) between the observations and model predictions, are used to measure the model prediction performance. The values of the three metrics, over the training, validation and test data, are shown in Table 2.

For comparison purposes, we made a change to the model settings described at the beginning of this section, where the =0 maximum time lag p =8 was changed to p=0, meaning that autoregressive variables such as AMOC(t-1) are not used for model building. The resulting model contains a total of 13 terms. The values of three metrics, RMSE, MAE and CC, for the 13-term model were put in Table 2. It can be observed that the model with the lagged autoregressive variables of the AMOC obviously outperforms the model without using autoregressive variables.

Finally, to further evaluate the model prediction quality, the 95% prediction confidence interval (i.e., at the 0.05 significance level) is depicted in Figure 5.

Tab. 1. The identified model for the AMOC.

Index	**Model Term**	**Parameter**	[a] **Contribution (%)**
1	AMOC(k-1)	0.6022	38.1072
2	NAO(k-1) × NAO(k-1) × U(k-7)	0.6916	8.7221
3	V(k-3) × AMOC(k-3)	0.4592	6.0087
4	AMOC(k-1) × AMOC(k-4)	-0.0497	3.9661
5	NAO(k-1) × NAO(k-4)	-0.6829	3.8206
6	NAO(k-3) × NAO(k-7) × U(k-6)	1.0087	2.2903
7	NAO(k-1) × NAO(k-3) × NAO(k-6)	0.6571	3.4517
8	NAO(k-6) × AMOC(k-1)	0.1628	2.3487
9	V(k-8) × AMOC(k-1) × AMOC(k-4)	-0.0783	1.8363
10	NAO(k-7) × U(k-3) × AMOC(k-3)	-0.2651	1.6454
11	NAO(k-3) × V(k-3)	0.8295	2.1726

[a] The contribution (in percentage) made by the model term to explaining the change in the response variable.

Tab. 2. The model performance, measured by RMSE, MAE and CC.

[a] **Model**	**RMSE**			**MAE**			**CC**		
	Training	Validation	Test	Training	Validation	Test	Training	Validation	Test
1	1.92	1.79	2.00	1.42	1.36	1.68	0.85	0.61	0.63
2	2.19	1.89	2.12	1.59	1.59	1.80	0.81	0.66	0.57

[a] Model 1 – As shown in Table 1; Model 2 – No lagged autoregressive variables is used for model building.

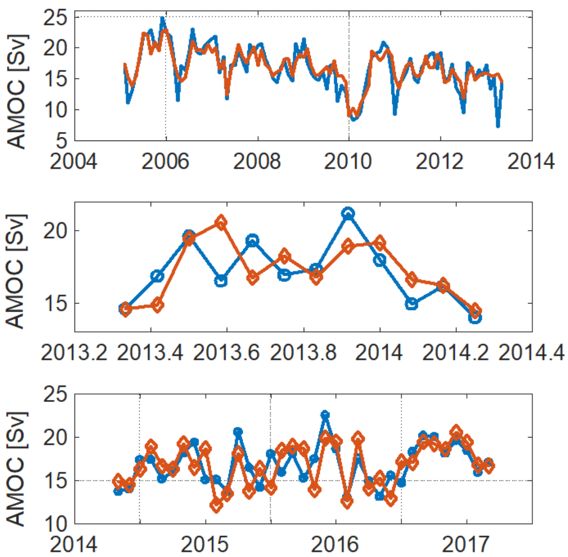

Fig. 4. An illustration of the comparison between one-month ahead predictions (from the model given in Table 1) and the corresponding observations, over the training data (April 2004-March 2013), validation data (April 2013 – March 2014) and test data (April 2014 – February 2017). Blue curve – observations; Red curve – one-month ahead prediction.

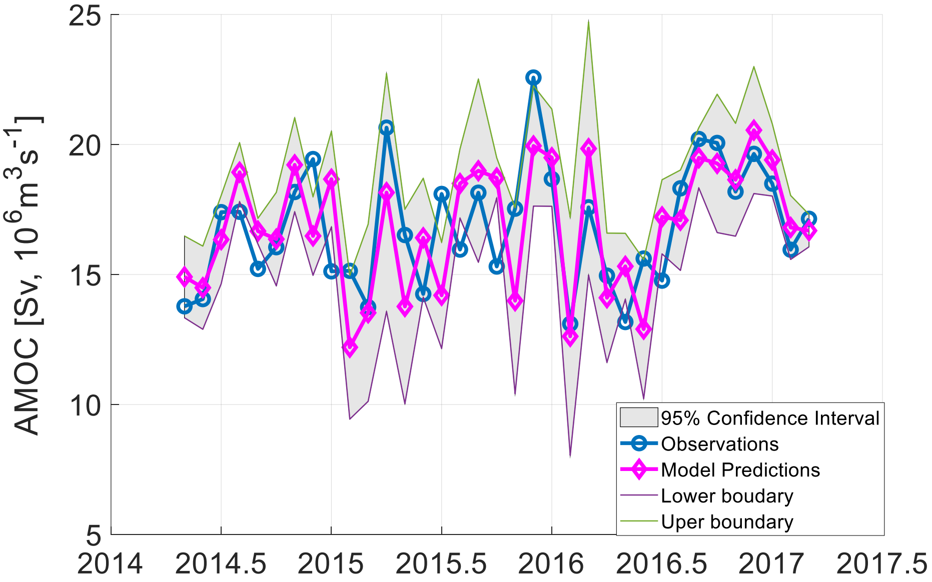

Fig. 5. The one-month ahead prediction of the AMOC at the 0.05 significance level, over the test data (April 2014 – February 2017) using the model reported in Table 1.

Conclusion

This paper focuses on AMOC modelling and prediction by addressing the following two questions: 1) How do the NAO index, the mean of the density variables U, the meridional density difference variable V, collectively and interactively affect the variability of the AMOC? 2) Can these three drivers, NAO, U and V, be used to build a good transparent predictive model for the AMOC?

For the first question, we carried out a case study on the monthly AMOC data measured during April 2004 – February 2017, and applied a nonlinear system identification method and a TIPS (transparent, interpretable, parsimonious and sparse) model, called NARMAX, to the AMOC data and the associated monthly data of NAO, U and V. A nonlinear model that well represents the dynamic relationship from the three drivers to the AMOC was developed from these data. The model suggests that the autoregressive variable AMOC(*t*-1) may play a significant role in explaining the near-term future (one month ahead) variability of the AMOC process. Other two autoregressive variables AMOC(*t*-3) and AMOC(*t*-4), together with the input lagged variables of NAO, U and V, collectively and interactively drive the behaviour of the AMOC process. For the second question, the model prediction results suggest a clear and positive answer.

The study concentrates on the development of TIPS models but does not explore the applicability of other complex machine learning methods. This is because unlike the NARMAX model which builds white-box models for complex black-box systems, most other machine learning methods, especially deep neural networks, are black-box models which are opaque to end users, and more than often even the model developers themselves cannot clearly 'see' what happens inside the neural network models and how the predictions are derived. Additionally, the training of complicated neural network models usually needs a sufficiently large number of samples, and such a requirement may make complex neural networks unsuitable or inapplicable for the smaller-sample-size AMOC modelling task here. However, complicated neural networks are powerful for learning nonlinear relationships from data, so therefore in future we will carry out feasibility studies on the applicability of other machine learning methods, to investigate the possibilities to further improve the prediction of the AMOC.

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We acknowledge that monthly, standardized and normalized, values of the NAO index were taken from https://psl.noaa.gov/data/ or https://psl.noaa.gov/gcos_wgsp/Timeseries/.

We acknowledge that the data of density anomalies at Gulf of Mexico (GM), Labrador Sea (LS) and Norwegian Sea (NS) were sourced from http://www.cpc.ncep.noaa.gov/products/GODAS/pl/introduction_godas_web.pdf).

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