

Research and Solution Proposals to Optimize Distribution Power Grids in Smart Grid Condition

PHAM Trung Son^{1,*}, NGUYEN Dinh Tien², NGUYEN Quang Thuan³, DANG Quang Khoa⁴

¹ Hanoi University of Mining and Geology, 18 Vien street, Hanoi, Vietnam

² Hanoi University of Industry, Hanoi, Vietnam

³ Thuyloi University, Hanoi, Vietnam

⁴ Vinh University of Technology Education, Nghe An, Vietnam

Corresponding author: phamtrungson@humg.edu.vn

Abstract. Smart Grid is a concept for transforming the electric power grid by using advanced automatic control and communications techniques and other forms of information technology. It integrates innovative tools and technologies from: generation, transmission and distribution. This also includes consumer appliances and equipment. This concept integrates energy infrastructure, processes, devices, information and markets into a coordinated and collaborative process. All allowing energy to be generated, distributed and consumed flexibly and efficiently. However, the Smart Grid with the integration of distributed generation itself also creates a several disadvantages. There can be problems with: stability and reliability, relay protection, isolation and operational isolation in which the problem is to create a burden on the distribution grid when transmitting electrical energy sources. Optimizing power flow and bringing high operating efficiency on Smart Grid conditions is an urgent issue. This paper focuses on researching and proposing solutions for optimal calculation of power flow on Smart Grid. The paper has researched, and analyzed calculation solutions to optimize power flow and proposed to use the Lagrange multiplier method. The study performed calculations for a typical Smart Grid model with three distributed generations. Calculation results have shown that the role of the method is to fully perform the optimal calculation of the power flow on the grid. This is in order to reduce power loss and energy loss as well as increasing operational efficiency while improving power quality in Smart Grid conditions.

Keywords: Smart grid, Power loss, Energy loss, Optimization, Distribution power grids

1. Introduction

The term Smart Grid refers to a modernization of the electrical network consisting of the integration of various technologies such as dispersed generation (DG), dispatchable loads, communication systems and storage devices utilized to efficiently deliver sustainable, economic and secure electricity. The Smart Grid concept is naturally associated with the integration of significant levels of Distributed Energy Resource (DER), including DGs, Demand Response (DR) [1, 2, 3], energy storage devices, and other energy sources into the electric grid. The Smart Grid scenario uses a two-way flow of electricity and information between power network and consumers in order to create an automated and widely distributed energy delivery network. The most important features of Smart Grid are the following: increasing the integration of renewable resources; increasing the participation of consumers in the network operation; decreasing the transmission and distribution losses, and lowering the energy cost for customers, subsequently; decreasing the electrical power consumption, and the emission of fossil fuels, concurrently; enabling consumers and electrical companies to control the demand.

Smart Grid or microgrid drivers are great in number, and linked to various factors. Such factors include: the necessity of controlling dispersed generation, ensuring power supply in remote areas, improving demand-side management, increasing energy efficiency, and creating self-healing electrical networks. The main objectives of Smart Grids are to increase supply reliability and improve power system security against contingencies or malicious attacks. Another important driver towards implementing Smart Grid is the higher integration of renewable energy resources on the distribution system level-which has substantially increased control problems in these systems [1, 4, 5].

Smart microgrids have served an integral role in the evolution of the Smart Grid [1, 6]. From this perspective, the Smart Grid can be divided into a system of integrated smart microgrids [1, 7, 8, 9]. In fact, the smart microgrid can be considered and exploited as the main building block of the smart grid [1, 10]. Therefore, Smart Grids and smart microgrids share common aspects such as interconnection with utility, interruptible loads, the use of different sources, employment of energy storage devices, optimal control based on customer requirements, optimal operation, the use of communications bandwidth for fast

applications and GPS, time-tagging, and cyber security [1, 11]. Based on these considerations, it follows that facing the problems of Smart Grids, and particularly smart microgrids, there should be revisions in traditional power system studies.

In current power systems, electrical losses are significant in the distribution of electrical energy, especially at lower voltage levels. Loss reduction can be achieved through the optimal control of power flow on the grid and the appropriate control of Distributed Generation (DG) resources in the distribution systems [1, 12], or more generally, through the control of dispatchable resources (DG, load, storage), which can be effectively assessed by using tools such as optimal power flow-like software. Connecting the new power sources to the current distribution systems leads to some technical and economic challenges [1, 13]. A possible solution for modern distribution systems consists of the creation of more or less independent cells which can interact in an internet-like structure. Microgrids can constitute the single element of this cellular structure in a large interconnected power system or be the natural answer to power supply in remote areas. In this regard, considering multi-microgrids as a system of microgrids would lead to different economic effects on the future Smart Grids [1, 14]. Also, preserving privacy of Optimal Power Flow models in this system is an important aspect which is discussed in [1, 15].

Operating the power transmission system when there is a high proportion of DG is really difficult and challenging. Most green energy sources rely on uncontrollable resources. In other words, generating electricity from renewable energy comes from natural resources such as sunlight, wind, or ocean waves. These types of sources generate intermittent and unstable power. Integrating large amounts of power from these types of sources- that cannot be regulated in a power supply system is a challenging task.

Optimizing power flow, bringing high operating efficiency under Smart Grid conditions is one of the urgent problems that need to be solved. The following content of the paper focuses on researching and proposing solutions for optimal calculation of power flow in order to reduce power loss and energy loss, increasing the improvement of operational efficiency while improving power quality in Smart Grid conditions.

2. Deterministic Optimal Power Flow

Power flow studies are necessary for planning, operating, and economic scheduling. This also includes other analysis; such as transient stability, voltage stability, and contingency studies. The tasks of power flow calculation are to solve the steady-state operating conditions of power systems based on the operation modes and the system's wiring.

Optimization of power flow is the task of identifying the optimal process from among others, compared by the criterion of optimality. Optimization can be distinguished as:

1. Determination of the optimal strategy for the development of power systems - construction or reconstruction of power systems and individual facilities (selection of location and capacity, setting the dates for commissioning new power plants, substations and transmission lines;
2. Selection of the best configuration of electrical networks;
3. Distribution of loads between individual power plants of an operating or projected system;
4. Choosing a strategy for the best use of material resources (types of fuel, etc.);

Therefore, the optimal power flow is necessary to minimize cost and energy losses ensuring increased revenue. The optimal power flow is a nonlinear characteristic optimization problem in a power system. The optimal power flow aims to minimize the total production cost by satisfying the load demand and system equality and inequality constraints [16, 17, 18]. The objective of optimal power flow is to find the setting of control variables at which objective function is optimized. Some of the possible objective functions in optimal power flow are power generation cost minimization, power transmission loss minimization, fuel emission during power generation, and minimum shift of generation from some optimum operation point [16, 19].

To solve the optimal power flow problem, many conventional and computer algorithms developed like lambda iteration method, base point, participation factors method, quadratic programming, nonlinear programming algorithm, gradient method Genetic Algorithm (GA), Evolutionary Programming (EP) [16, 20]. Optimal power flow has been used widely in planning and operation. Previously, several classical and modern optimization techniques have been proposed to solve the optimal power flow problem [16, 21] such as Particle Swarm Optimization (PSO) algorithm, Artificial Bee Colony (ABC) algorithm [16, 22], Firefly algorithm (FFL), Grasshopper Optimization Algorithm (GOA) [16, 23], Dynamic Programming (DP) [16, 24], Modified algorithm for finding the optimal nod of closed-medium voltage grids [25], Loop cutting algorithm [26].

According to the above discussion in optimal power flow, the objective is to optimize the power flow,

along with a satisfying a number of constraints. First, power flow studies should be should be thoroughly understood and then in power flow studies, planning for the future load, economic scheduling, and control of existing and future power systems should be designed.

This paper presents the solution of optimal power flow on Smart Grid using the Lagrange multiplier method. The proposed Lagrange multiplier method is employed for power loss optimization, which is a very critical issue in optimal power flow. Power loss across transmission lines is a major concern in the recent electrical power crisis. Therefore, the scope of this work is to minimize the power loss effectively and to evaluate the performance and analyze the effective optimization of the Lagrange multiplier method.

2.1 Mathematical formulation of the problem of optimization of the regime of electric power systems

The optimization of the regime of electric power systems can be formulated mathematically as follows [27, 28, 29]. There is a function of n variables - $F(x_1, x_2, \dots, x_n)$. These variables are related to each other by k equations or constraint inequalities:

$$\left. \begin{matrix} W_1(x_1, x_2, \dots x_n) \geq 0; \\ W_2(x_1, x_2, \dots x_n) \geq 0; \\ \dots \\ W_k(x_1, x_2, \dots x_n) \geq 0. \end{matrix} \right\} \quad (1)$$

where W_1, W_2, \dots, W_k are some functions of the variables x_i ($i = 1,2, \dots n$). It is required to find the minimum of the function F . Solving the optimization problem under constraints in the form of inequalities requires the use of very complex optimization methods (Kuhn-Tucker method, etc.). We will consider simpler optimization methods with variable constraints in the form of equations. In this case, the number of equations k must be less than n.

2.2 Lagrange multiplier method

When solving problems of regime optimization, the method of indefinite Lagrange multipliers is widely used. In this case, instead of the conditions for the extremum of the function $F(x_1, x_2, \dots x_n)$ n variables related to each other by k relations (1), one looks for the conditions for the extremum of the Lagrange function [27, 28, 29]:

$$S = F + \sum_{i=1}^k \lambda_i W_i \quad (2)$$

where λ_i ($i = 1,2, \dots k$) are constant factors determined when finding the function F . These factors are called indefinite Lagrange multipliers.

Equating to zero, the partial derivatives of S with respect to all n variable functions, we obtain the following n equations:

$$\left. \begin{matrix} \frac{\partial S}{\partial x_1} = \frac{\partial F}{\partial x_1} + \sum_{i=1}^k \lambda_i \frac{\partial W_i}{\partial x_1} = 0 \\ \frac{\partial S}{\partial x_2} = \frac{\partial F}{\partial x_2} + \sum_{i=1}^k \lambda_i \frac{\partial W_i}{\partial x_2} = 0 \\ \dots \end{matrix} \right\} \quad (3)$$

From n equations (1) and k constraint equations (1), we compose a total of (n + k) equations. The number of unknowns is also equal to (n + k), namely: n desired values of the variables - $x_1, x_2, \dots x_n$ - and k Lagrange multipliers - $\lambda_1, \lambda_2, \dots \lambda_k$.

This makes it possible to find the arguments corresponding to the extremum of the function S . But the same values, as is known, characterize the extremum of the minimized function F .

In the considered method, the arguments corresponding to the extremum of the minimized function F were determined. In order for the extremum found to really be a minimum, it is necessary to check the sign of the second differential of the functions F or S . If $d^2F > 0$ or $d^2S > 0$, then this extremum is a minimum. Determining the sign of d^2F or d^2S is very difficult. Based on experience, one proceeds from a number of assumptions that allow us to consider that the found extremum is a minimum.

2.3 Application of the Lagrange multiplier method to optimize power flows on the small Smart Grid

Let's find the power flow in the grid in Figure 1, corresponding to the smallest active power losses. The diagram consists of three nodes: one source node (DG) and two load nodes.

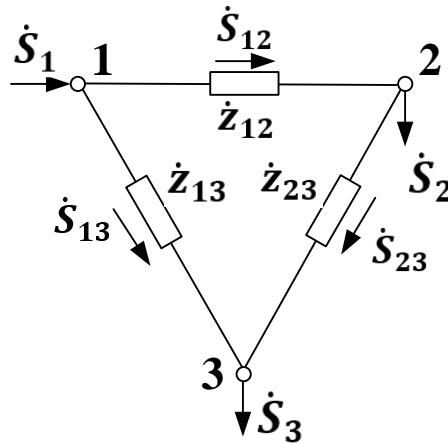


Fig. 1. Simple closed grid diagram.

Application of the Lagrange multiplier method to solve the problem of optimal distribution of power flows in the network consists in determining the minimum of the Lagrange function, which includes the active power losses:

$$\min \Delta P = \min \left(\frac{P_{12}^2 + Q_{12}^2}{U_{\text{norm}}^2} r_{12} + \frac{P_{23}^2 + Q_{23}^2}{U_{\text{norm}}^2} r_{23} + \frac{P_{13}^2 + Q_{13}^2}{U_{\text{norm}}^2} r_{13} \right) = \min \sum \frac{P_{kj}^2 + Q_{kj}^2}{U_{\text{norm}}^2} r_{kj} \quad (4)$$

and the equations of the first Kirchoff law:

$$\begin{cases} P_{12} - P_{23} = P_2; \\ P_{13} + P_{23} = P_3; \\ Q_{12} - Q_{23} = Q_2; \\ Q_{13} + Q_{23} = Q_3. \end{cases} \quad (5)$$

each of which is multiplied by the corresponding Lagrange multiplier. Consider the problem of optimizing the grid mode in Fig. 1, when the fluxes of reactive power in the lines Q_{kj} are equal to zero.

The equality of flows Q in lines 12, 23, 31 to zero means that at nodes 2 and 3 in Figure 1 full compensation of reactive power takes place. It is necessary to define

$$\min \Delta P = \min \left(\frac{P_{12}^2}{U_{\text{norm}}^2} r_{12} + \frac{P_{13}^2}{U_{\text{norm}}^2} r_{13} + \frac{P_{23}^2}{U_{\text{norm}}^2} r_{23} \right) \quad (6)$$

under the two equality constraints from (5)

$$\begin{cases} P_{12} - P_{23} = P_2 \\ P_{13} + P_{23} = P_3 \end{cases} \quad (7)$$

Lagrange function

$$F = \frac{P_{12}^2}{U_{\text{norm}}^2} r_{12} + \frac{P_{13}^2}{U_{\text{norm}}^2} r_{13} + \frac{P_{23}^2}{U_{\text{norm}}^2} r_{23} + \lambda_1 (P_{12} - P_{23} - P_2) + \lambda_2 (P_{13} + P_{23} - P_3) \quad (8)$$

where λ_1 and λ_2 are the Lagrange multipliers.

The conditional extremum problem (6), (8) with three variables P_{12} , P_{23} and P_{13} is reduced to the definition of an unconditional extremum (minimum) of the Lagrange function, which depends on five variables; three power flows and two Lagrange multipliers λ_1 and λ_2 . The minimum of the Lagrange function corresponds to the solution of the original problem and is determined by the equality to zero of five partial derivatives (equation 9).

To solve the system of linear algebraic equations (9), we transform its first three equations into the equation of the second Kirchoff law, excluding the Lagrange multipliers from them. As a result, we get the expression 10.

$$\left. \begin{aligned} \frac{\partial F}{\partial P_{12}} &= \frac{2P_{12}r_{12}}{U_{\text{norm}}^2} + \lambda_1 = 0 \\ \frac{\partial F}{\partial P_{23}} &= \frac{2P_{23}r_{23}}{U_{\text{norm}}^2} - \lambda_1 + \lambda_2 = 0 \\ \frac{\partial F}{\partial P_{13}} &= \frac{2P_{13}r_{13}}{U_{\text{norm}}^2} - \lambda_2 = 0 \\ \frac{\partial F}{\partial \lambda_1} &= P_{12} - P_{23} - P_2 = 0 \\ \frac{\partial F}{\partial \lambda_2} &= P_{13} + P_{23} - P_3 = 0 \end{aligned} \right\} \quad (9)$$

$$\frac{P_{12}}{U_{\text{norm}}^2} r_{12} + \frac{P_{13}}{U_{\text{norm}}^2} r_{13} + \frac{P_{23}}{U_{\text{norm}}^2} r_{23} = 0 \quad (10)$$

Solving the last two equations of system (9) together with this equation, we obtain

$$P_{12}r_{12} + (P_{12} - P_2)r_{23} + (P_{12} - P_2 - P_3)r_{13} = 0 \quad (11)$$

from here determine the optimal power flow running on the line

$$P_{12} = \frac{P_2(r_{23}+r_{13})+P_3r_{13}}{r_{12}+r_{23}+r_{13}} \quad (12)$$

From the formula, it is possible to adjust the optimal power flow on the line with any variation of the load and of the power supply. Through intelligent monitoring and control devices of Smart Grid, any changing working modes on the grid can be adjusted in a timely manner, keeping the mode parameters at an optimal level.

2.4 Test diagram used to calculate the optimal power flow

As examples of the application of the Lagrange multiplier method to optimize the power flow. The test diagram and detail parameters in Smart Grid condition are shown in Figure 2.

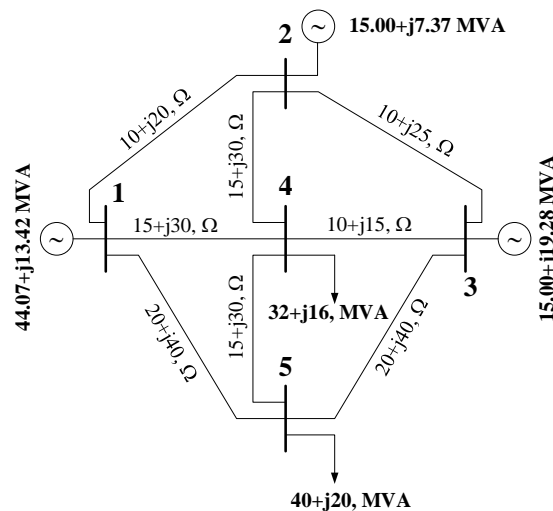


Fig. 2. The test calculation diagram.

Apply Lagrange multiplier method. The optimal power flow on the network segments is calculated and shown right on the Figure 3. The results of the optimal calculation of the power flow of the diagram are detailed in the Table 1.

The total active power loss on the grid is 2.07MW, the total active power transmission on the grid is 98.98MW. The optimal active power loss ratio is 2.09%. Thus, with the existing scheme, operating costs in the optimal mode of power flow will be reduced to the lowest level compared to other operating modes.

Through the optimal calculation results, corresponding to each operating mode, it is possible to determine the required power flow for transmission so that the loss on all lines is the lowest. Therefore, contributing to improving transmission efficiency, ensuring power quality while minimizing power system operating costs.

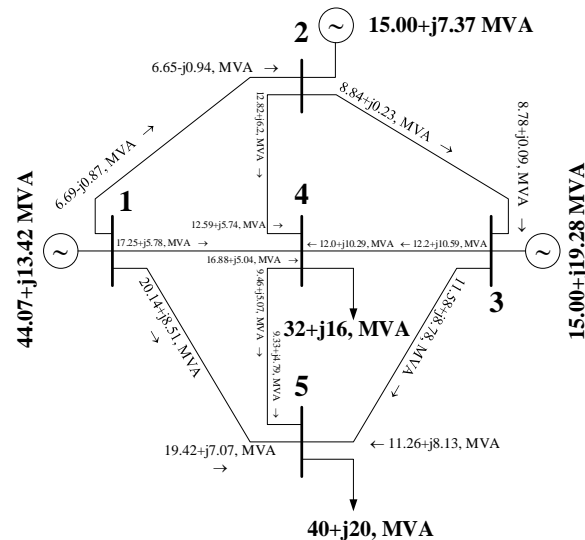


Fig. 3. Calculation results of the test diagram.

Tab. 1. Optimal calculation results of the test calculation diagram.

| Network segment | Power at the beginning of the line, MVA | Power at the end of the line, MVA | Loss, MVA |
|-----------------|---|-----------------------------------|------------|
| 1-2 | 6.69-j0.87 | 6.65-j0.94 | 0.04+j0.07 |
| 1-4 | 17.25+j5.78 | 16.88+j5.04 | 0.37+j0.74 |
| 1-5 | 20.14+j8.51 | 19.42+j7.07 | 0.72+j1.44 |
| 2-3 | 8.84+j0.23 | 8.78+j0.09 | 0.06+j0.14 |
| 2-4 | 12.82+j6.2 | 12.59+j5.74 | 0.23+j0.46 |
| 3-4 | 12.2+j10.59 | 12.0+j10.29 | 0.2+j0.3 |
| 3-5 | 11.58+j8.78 | 11.26+j8.13 | 0.32+j0.65 |
| 4-5 | 9.46+j5.07 | 9.33+j4.79 | 0.13+j0.28 |

The Smart Grid with the integration of distributed generation itself also creates several disadvantages. There can be problems with stability and reliability, relay protection, isolation and operational - in which the problem is to create a burden on the distribution grid when transmitting electrical energy sources. Therefore, optimizing power flow will bring high operating efficiency.

Smart Grid with the integration of distributed generation have unstable power sources, which cause power imbalances at nodes, which can result in large power losses and energy losses. It does not guarantee operational efficiency and leads to great economic loss. Thanks to the optimal calculation solution for the power flow, corresponding to any operating mode, Smart Grid's intelligent dispatching system -which uses the optimal calculation results of the power flow- will control it in time, so that the operating mode is consistently optimal.

Innovative Smart Grid solutions such as storage, will ensure stable power generation of distributed generations. The optimal power flow on the grid is determined by the optimal calculation through the Lagrange multiplier method, which will optimize the operating moderation.

3. Conclusions

The current energy transition is characterized by the desire to transition from traditional fossil fuels to more sustainable sources of electricity production. This transition necessitates the integration of new technologies such as renewable energy generators combined with the application of technology, equipment measurement, monitoring, control, and modern tools of communication. New electric production technologies have presented new design and implementation challenges due to the specificity of each resource. Traditional planning and operational strategies have solved these types of problems in the past by infrastructure reinforcements of the transmission or distribution grid. Innovative Smart Grid solutions such as storage, demand side management and curtailment were proposed as possible alternatives for infrastructure investments. These solutions require sophisticated power flow analysis tools. Existing power

analysis tools were presented including power flow analysis and optimal power flow analysis. Optimal power flow analysis was described in more detail to justify the choice of this tool as opposed to power flow analysis. These sophisticated algorithms present new mathematical challenges as well, which were discussed along with proposed solutions to overcome these challenges. This paper has presented work contributing to the advancement of power system analysis for Smart Grids in the presence of high renewable energy penetration. The solution used to calculate the optimal power flow is the Lagrange multiplier method. New sophisticated planning and operational tools are necessary to analyze the effects of high distributed generation penetration on existing architecture. The optimization of power flow of existing networks through automation and control has been discussed.

The Lagrange multiplier method was successfully designed and implemented to solve the optimal power flow problem. The comparison of results for the test diagram clearly shows that the Lagrange multiplier method is indeed capable of obtaining optimum solution efficiently for power flow problems. After finding the solution, the line losses have also been reduced by regulating the power flow on the grid. This indicates the significance of the Lagrange multiplier method to solve optimal power flow problems interconnected power system network.

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