



# Application of Robust Estimation Method for Establishing 3D Combined Terrestrial and GNSS Network: A case of a Quarry in Lang Son, Vietnam

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## Abstract

Recently, in Vietnam, the detection of geodetic measurements that contain rough errors as well as such data processing method has been considered as a key step in geodetic data processing, especially for large geodetic networks with many different types of measurements like 3D - Global Navigation Satellite Systems (GNSS) network. On the other hand, mines in Vietnam often have complex terrains, so it is necessary to apply modern and flexible surveying methods in combination with ground and space measurements to build 3D coordinates control networks for management and exploitation to ensure sustainable development. Therefore, this research developed a Robust estimation method based on empirical weighting function for establishing 3D geodetic network combining terrestrial observation and GNSS vectors. The experiment on processing the combined network in Lang Son limestone quarry, Vietnam showed that the proposed method could be an effective solution for processing 3D terrestrial - GNSS geodetic network for mine surveying in Vietnam.

**Keywords:** robust estimation, geodetic network, network adjustment, least-square

## 1. Introduction

Currently, as Vietnam is modernizing mining, the speed and volume of mineral exploitation are increasing in a wide range. Therefore, it is essential to evaluate the impacts of mineral exploitation affecting mine safety as well as to manage and forecast mineral deposits. In addition, the terrain of mine area is specific and complex, so it is necessary to study methods of building modern geodetic grids to meet multiple objectives such as monitoring of mine transfer, calculating exploitation volume, calculating mineral deposits, exploitation planning, etc.

The strong development of measurement technology with many modern high-precision measuring devices such as Global Navigation Satellite Systems (GNSS), total stations, etc. has contributed an effective solution in the construction of geodetic network, especially geodetic network for mining. However, each technique has its own drawbacks, in order to overcome the disadvantages of each measurement method, a common solution is to combine GNSS technology and traditional measurement technology to form a 3D terrestrial - GNSS geodetic network with high flexibility and accuracy (Krakiwsky and Thomson, 1974; Thomson 1976; Groten, 1977).

Geodetic data processing in general or adjustment of a geodetic network in particular is one of the important tasks in surveying. When constructing geodetic networks, measurement errors are inevitable. Therefore, network adjustment is required to find the most reliable values of unknown quantities. In this processing, rough error adversely affects the results of the adjustment problem. Modern adjustment theory has been studying the impact of rough errors on post-adjustment results and how to deal with them. Geodetic data obtained through statistics and analysis show that the prob-

ability of appearing rough errors accounts for about 1% ÷ 10% (Tukey, 1962). One of the effective methods to handle rough errors is a robust estimation (Huber, 1964, 1981; Hampel, 1986; Koch, 2013, 2014; Koch and Kargoll, 2013). Some scientists formulated weighted functions or various robust estimation methods such as the Danish method, the Tukey method, and the L1-norm method. However, the above methods consider the measurements to be independent and do not consider the correlation of the measurements in a geodetic network, especially the GNSS geodetic network.

Therefore, in this study, we use the Huber method and the weight function equivalent to two coefficients developed by Yang Y, et al. (2002) to develop a robust estimation with the application of empirical weight function to process 3D terrain - GNSS geodetic network data in order to meet the requirement of mine surveying in Vietnam.

## 2. Literature review of robust estimation

### 2.1 Huber method

The Huber method was proposed by (Huber, 1981), and its function is expressed as follows:

$$\rho(v) = \begin{cases} \frac{1}{2}v^2 & |v| \leq k \\ k|v| - \frac{1}{2}k^2 & |v| > k \end{cases} \quad (1)$$

where,  $v$  is the correction of the observation calculated from the previous iteration;  $k$  is constant, it is possible to take  $k = (2\sigma + 3\sigma)$ , and the corresponding weighting factor is

$$w_i = \begin{cases} 1 & |v_i| \leq k \\ \frac{k}{|v_i|} & |v_i| > k \end{cases} \quad (2)$$

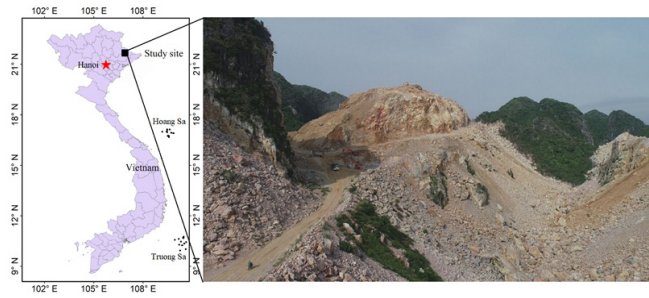


Fig. 1. Location and landscape of Lang Son quarry  
Rys. 1. Położenie i krajobraz kamieniołomu Lang Son

Tab 1. Terrestrial edge measurements  
Tab. 1. Naziemne obserwacje

| No. | Name of edge   |              | Length (m) |
|-----|----------------|--------------|------------|
|     | Starting point | Ending point |            |
| 1   | I              | V            | 357.225    |
| 2   | I              | III          | 934.485    |
| 3   | I              | IV           | 647.881    |

## 2.2 Tukey method

According to Tukey (1962), Tukey function has the form as follows:

$$\rho(u) = \begin{cases} \frac{1}{6}[1-(1-u^2)^3] & |u| \leq 1 \\ \frac{1}{6} & |u| > 1 \end{cases} \quad (3)$$

$$\varphi(u) = \begin{cases} u(1-u^2)^2 & |u| \leq 1 \\ 0 & |u| > 1 \end{cases} \quad (4)$$

Tukey's weighting function is as follows

$$w(u) = \begin{cases} (1-u^2)^2 & |u| \leq 1 \\ 0 & |u| > 1 \end{cases} \quad (5)$$

where  $u=v/c$ .MAD,  $c$  is the regression coefficient.

## 2.3. Danish method

The principle of the Danish method is based on the indication of outliers by corresponding major corrections. After least squares adjustment, the priority weights of measurements are replaced by correction function. According to Kraup and Kubik (1983), the weighting function is as follows:

$$w(v_i) = \begin{cases} 1 & |v_i| \leq c \\ \exp\left(-\frac{v_i^2}{c^2}\right) & |v_i| > c \end{cases} \quad (6)$$

where  $c$  is constant and is often chosen as follows:

$$c = 1.5 \hat{\sigma}$$

## 2.4 L1 - norm method

The L1-norm is one of the most successful robust estimation (Harvey, 1993) and is expressed as follows (Wang et al., 2006):

$$\rho(u) = |u| \quad (7)$$

In L1-method, the corresponding weighting factor is

$$w_i = \frac{\varphi(v_i)}{v_i} = \frac{\partial |v_i|}{\partial v_i} \cdot \frac{1}{|v_i|} = \frac{1}{|v_i|} \quad (8)$$

To solve the problem of weighting when  $v_i=0$ , the weighting factor can be taken as

$$w_i = \frac{1}{|v_i|+k} \quad (9)$$

where  $k$  is a small number. The principle of correction is

$$\sum_{i=1}^n p_i |v_i| = \min \quad (10)$$

On the other hand, with  $p_i=p_i w_i$  to be replaced in  $V = AX + L$  to gain a standard equation system and its solution  $X$  is determined by the formula (11) as follows:

$$A^T \bar{P} A X + A^T \bar{P} L = 0 \quad (11)$$

$$X = -(A^T \bar{P} A)^{-1} A^T \bar{P} L$$

The above robust estimation methods consider the measurements to be independent and generally apply to small geodetic grids when the measurements have the same accuracy. However, for a large geodetic network with many types of measurement of different accuracy, especially 3D-GNSS geodetic networks, the correlation factor between measurements should be taken into account so that the above methods might be not effective

## 3. Robust estimation for 3D Terrestrial – GNSS network

To overcome the disadvantages of the Huber, Tukey, L1 - norm and Danish robust estimation method when applying for the 3D terrain – GNSS geodetic network, the robust estimation method for the measurements not of the same accuracy is used according to (Wang et al., 2006):

Assuming that error equation is as following

$$V_{n \times 1} = A_{n \times n} \hat{X}_{n \times 1} + L_{n \times 1} = \begin{bmatrix} a_1 \\ a_2 \\ \dots \\ a_n \end{bmatrix} \begin{bmatrix} \hat{X}_1 \\ \hat{X}_2 \\ \dots \\ \hat{X}_n \end{bmatrix} + \begin{bmatrix} l_1 \\ l_2 \\ \dots \\ l_n \end{bmatrix} \quad (12)$$

where  $A_{n \times n}$  is a coefficient matrix,  $a_i$  is the coefficient of matrix  $A$ ,  $X_{n \times 1}$  is unknowns vector and needed to be determined,  $L_{n \times 1}$  is

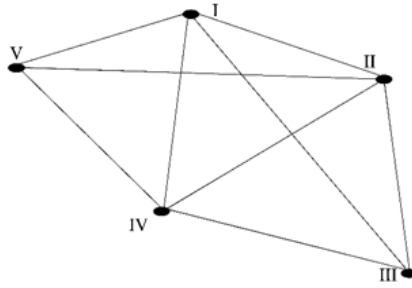


Fig. 2. Diagram of the experimental network  
Rys. 2. Schemat sieci eksperymentalnej

Tab. 2. GNSS observation in the geocentric coordinate system  
Tab. 2. GNSS obserwacje w geocentrycznym układzie współrzędnych

| No. | Information                           | Start point |                         | End point     |               |
|-----|---------------------------------------|-------------|-------------------------|---------------|---------------|
| 1   | BASELINE                              | V           |                         | IV            |               |
|     | $\Delta X \Delta Y \Delta Z$ (Global) |             | -295.006 m<br>93.389 m  | -439.121 m    |               |
|     | Aposteriori Covariance Matrix         |             | 0.0000025661            | -0.0000023862 | -0.0000010007 |
|     |                                       |             |                         | 0.0000055295  | 0.0000018945  |
|     |                                       |             |                         | 0.0000015047  |               |
| 2   | BASELINE                              | IV          |                         | III           |               |
|     | $\Delta X \Delta Y \Delta Z$ (Global) |             | -459.526 m<br>-2.803 m  | -169.063 m    |               |
|     | Aposteriori Covariance Matrix         |             | 0.0000017937            | -0.0000016789 | -0.0000004127 |
|     |                                       |             |                         | 0.0000088973  | 0.0000042289  |
|     |                                       |             |                         | 0.0000067321  |               |
| 3   | BASELINE                              | II          |                         | III           |               |
|     | $\Delta X \Delta Y \Delta Z$ (Global) |             | -106.968 m<br>209.755 m | -591.975 m    |               |
|     | Aposteriori Covariance Matrix         |             | 0.0000016524            | -0.0000017721 | -0.0000007093 |
|     |                                       |             |                         | 0.0000063326  | 0.0000023426  |
|     |                                       |             |                         | 0.0000032765  |               |
| 4   | BASELINE                              | II          |                         | V             |               |
|     | $\Delta X \Delta Y \Delta Z$ (Global) |             | 647.570 m<br>119.150 m  | 16.197 m      |               |
|     | Aposteriori Covariance Matrix         |             | 0.0000010863            | -0.0000010347 | -0.0000004184 |
|     |                                       |             |                         | 0.0000022376  | 0.0000008076  |
|     |                                       |             |                         | 0.0000007483  |               |
| 5   | BASELINE                              | II          |                         | IV            |               |
|     | $\Delta X \Delta Y \Delta Z$ (Global) |             | 352.564 m<br>212.549 m  | -422.920 m    |               |
|     | Aposteriori Covariance Matrix         |             | 0.0000027049            | -0.0000024597 | -0.0000010638 |

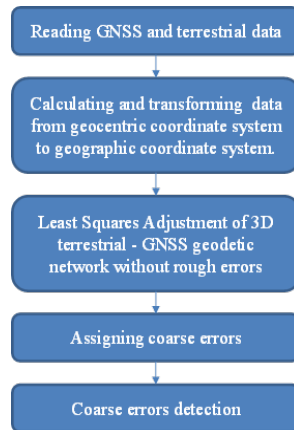


Fig. 3. Experimental flowchart  
Rys. 3. Schemat procesu eksperymentalnego

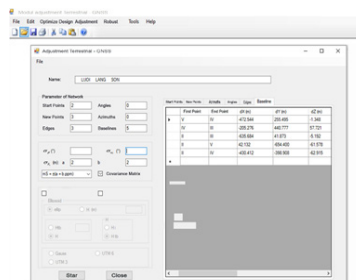


Fig. 4. Interface of combined network adjustment module  
Rys. 4. Interfejs modułu wyrównania sieci kombinowanej

Tab. 3. Deviations of correction after performing Robust estimation

Tab. 3. Odchylenia korekcji po wykonaniu estymacji odpornej.

| Robust estimation method | Deviation of correction<br>$\Delta v_i =  v_i - v_i^* $ (mm) |      |      |      |      |      |      |
|--------------------------|--|------|------|------|------|------|------|
|                          | 4σ   | 5σ   | 6σ   | 7σ   | 8σ   | 9σ   | 10σ  |
| Experimental function    | 9.5  | 13.4 | 16.8 | 20.2 | 23.4 | 26.4 | 29.6 |
| L1-norm                  | 7.3  | 9.2  | 14.9 | 17.7 | 20.1 | 24.3 | 26.3 |
| Tukey                    | 6.8  | 8.4  | 13.9 | 16.1 | 18.9 | 22.1 | 24.9 |
| Danish                   | 7.1  | 8.7  | 14.5 | 16.7 | 19.7 | 23.6 | 25.8 |
| Huber                    | 8.2  | 9.8  | 15.1 | 17.9 | 20.5 | 24.8 | 26.6 |

Tab. 4. Deviation of  $v$  to the assigned coarse error

Tab. 4. Odchylenie  $v$  od przypisanego błędu grubego

| Robust estimation method | Deviation of the $\Delta v$ to the assigned coarse error |     |     |     |     |     |     |
|--------------------------|--|-----|-----|-----|-----|-----|-----|
|                          | 4σ   | 5σ  | 6σ  | 7σ  | 8σ  | 9σ  | 10σ |
| Experimental function    | 2.5  | 1.6 | 1.2 | 0.8 | 0.6 | 0.4 | 0.4 |
| L1-norm                  | 4.7  | 5.8 | 3.1 | 3.3 | 2.7 | 3.7 | 3.7 |
| Tukey                    | 5.2  | 6.6 | 4.1 | 4.9 | 4.9 | 5.1 | 5.1 |
| Danish                   | 4.9  | 6.3 | 3.5 | 4.3 | 3.4 | 4.2 | 4.2 |
| Huber                    | 3.8  | 5.2 | 2.9 | 3.1 | 2.2 | 3.4 | 3.4 |

Tab. 5. Possibility to detect coarse errors of Robust estimation method Tabela 5. Możliwość wykrycia grubych błędów metody szacowania Robust

Tab.5. Możliwość wykrycia grubych błędów metodami szacowania estymacji odpornej

| Methods               | Percentage of observation containing coarse errors detected (%) |         |   |         |
|-----------------------|---|---------|---|---------|
|                       | 1 measurement containing coarse errors                          |         | 2 measurements containing coarse errors |         |
|                       | 3σ- 5σ  | 6σ- 10σ | 3σ- 5σ                                  | 6σ- 10σ |
| Experimental function | 83  | 98      | 81                                      | 97      |
| L1                    | 65  | 83      | 59                                      | 79      |
| Tukey                 | 61  | 70      | 50                                      | 63      |
| Danish                | 67  | 79      | 57                                      | 74      |
| Huber                 | 69  | 86      | 65                                      | 82      |

a free term matrix. Function  $\rho(l, X)$  in equation (1) of M estimation can be expressed as follows (Wang et al. 2006):

$$\rho(l, \hat{X}) = \rho(v_i) \quad (13)$$

As measurements are not accurate, the matrix of weight P is as in the following form

$$P_{mn} = \begin{pmatrix} p_1 & & & & \\ & p_2 & & & \\ & & \dots & & \\ & & & \dots & \\ & & & & p_n \end{pmatrix}_{mn} \quad (14)$$

Then the function  $\rho$  has the following form (Zhou et al., 1995; Wang et al. 2006):

$$\rho(v_i) = \frac{\partial \rho}{\partial v_i} \quad (15)$$

Take the derivative of expression (3) for X, symbolized as

$$\phi(v_i) = \frac{\partial \rho}{\partial v_i} \quad \text{and the result is} \quad (16)$$

$$\sum_{i=1}^n p_i \phi(v_i) \alpha_i = 0 \quad (16)$$

$$\sum_{i=1}^n \alpha_i^T \bar{p} v_i = 0 \quad \text{or} \quad A^T \bar{P} V = 0 \quad (17)$$

Replace  $V=AX+L$  in (17), then the standard set of equations of M estimation is

$$A^T \bar{P} A \hat{X} + A^T \bar{P} L = 0 \quad (18)$$

where: P is the equivalent weighting matrix,  $p_i$  is the equivalent weighting element,  $w_i$  is a weighting coefficient.

Parameter X of M robust estimation is determined as:

$$\hat{X} = -(A^T \bar{P} A)^{-1} \cdot A^T \bar{P} L \quad (19)$$

According to Huber (1964), the general form of is  $\phi(v_i)$ ,

$$\phi(w_i) \quad (20)$$

$$\phi(v_i) = \begin{cases} v_i & \text{for } |v_i| < c \\ \text{sign}(v_i)c & \text{for } |v_i| \geq c \end{cases} \quad (20)$$

$$\phi(w_i) = \begin{cases} 1 & \text{for } |w_i| < c \\ \frac{c}{|v_i|} & \text{for } |w_i| \geq c \end{cases} \quad (21)$$

where: c is the constant and normally takes the value of  $c=1.5 \div 2\sigma$

Independent measurements, the matrix of new weights is determined as:

$$\text{Set } \bar{p}_i = p_i w_i, \quad w_i = \frac{\phi(v_i)}{v_i} \quad (22)$$

Dependent measurements, the matrix of new weights is determined as [14]:

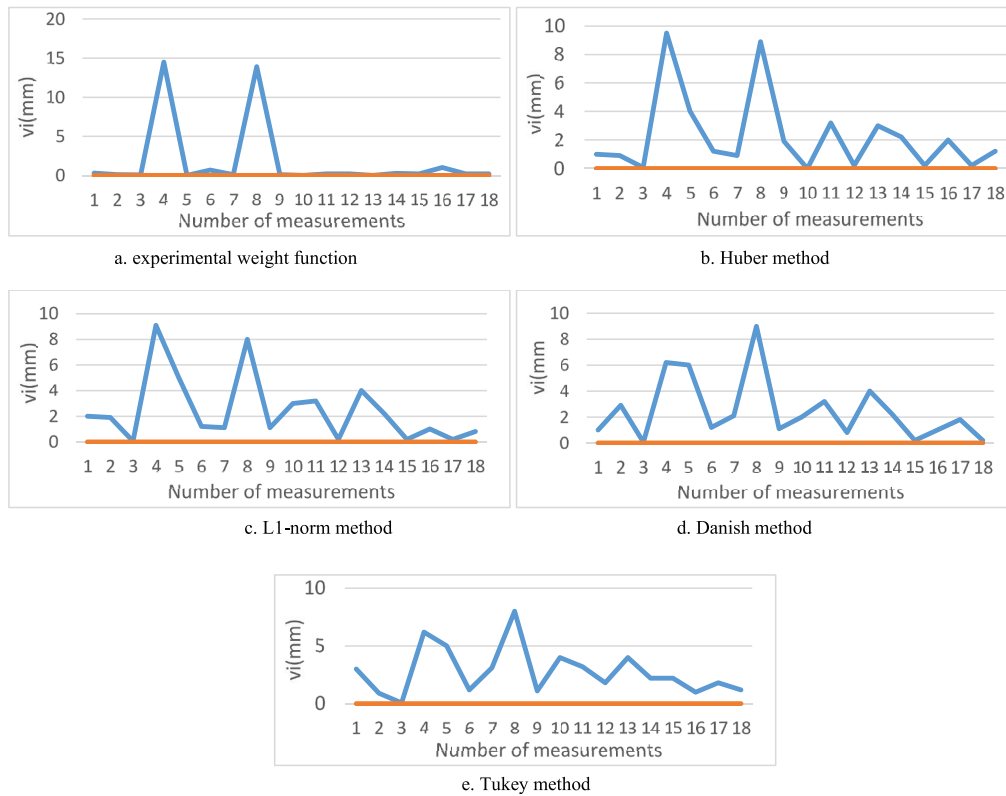


Fig. 5. Corrections by robust estimation methods in case 2 observations containing coarse error  
Rys. 5. Poprawki uzyskane po estymacji odpornej w przypadku 2 obserwacji zawierających błędy grube

$$\gamma_{ii} = \begin{cases} 1 & |\bar{v}_i| \leq c \\ \frac{c}{|\bar{v}_i|} & |\bar{v}_i| > c \end{cases} \quad (23)$$

$$\bar{p}_{ij} = p_{ij} \gamma_{ij} \quad (24)$$

$$\gamma_{ij} = \sqrt{\gamma_{ii} \gamma_{jj}} \quad (25)$$

where  $\gamma_{ii}$ ,  $\gamma_{jj}$  is the coefficient of weight loss  $\bar{v}_i = \frac{v_i}{m_{v_i}}$ .

However, the method of using the two-factor equivalent weighting function [14] is only used in the GNSS grid. Therefore, to process the data of the 3D terrestrial - GNSS geodetic network including both ground and satellite observation, the proposed method is based on the Huber weighting function and the two-factor equivalent weighting function [14] to investigate and select the parameters of the function for independent measurements, dependent measurements, and  $c$  coefficient. Through experiments, the empirical weighting function is introduced for 3D terrestrial - GNSS geodetic network as follows:

$$\begin{cases} w=1 & |\bar{v}| \leq c \\ w = \frac{c}{|\bar{v}|} \cdot \frac{v}{\sigma_{v_i}} \cdot \bar{p} = \bar{p} w; & |\bar{v}| > c \\ w = w_0 = 1 & |\bar{v}| \leq c < |\bar{v}_0| \\ w = \frac{c}{|\bar{v}|} \cdot \frac{v}{\sigma_{v_i}}; w_0 = \frac{c}{|\bar{v}_0|} \cdot \frac{v_0}{\sigma_{v_i}} \cdot \bar{p} = \bar{p} w_0; & |\bar{v}| > c < |\bar{v}_0| \end{cases} \quad (26)$$

where  $c$  is constant and can be chosen as  $c = 1.5$ ,  $\sigma_{v_i}$  is determined as the following formula:

$$\sigma_{v_i} = \sigma_0 \sqrt{(Q_{vv})_{ii}} \quad (27)$$

$$Q_{vv} = P^{-1} - A Q_x A^T \quad (28)$$

The Robust estimation includes four steps as follows:

Step 1:  $n$  values of the geocentric coordinate system were calculated and transferred into  $n$  values of the geographic coordinate system  $\Delta X$ ,  $\Delta Y$ ,  $\Delta Z$ . The GNSS geodetic network is formed by edge vectors according to the principle of relative positioning. Measurements in the GNSS geodetic network are components of edge vectors  $\Delta X$ ,  $\Delta Y$ ,  $\Delta Z$  determined in the geocentric system (WGS-84) incorporated with their covariance matrix (WGS-84) symbolized as  $C$ :

$$\begin{bmatrix} \Delta N \\ \Delta E \\ \Delta U \end{bmatrix} = R^T \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} \quad (29)$$

Simultaneously, the covariance matrix in the  $M$  system is determined based on the rotation matrix  $R$  and the covariance matrix in the geocentric system  $C_{XYZ}$ :

$$M = R^T C_{XYZ} R \quad (30)$$

In which:

$$C_{XYZ} = \begin{bmatrix} Var(X) & Cov(X,Y) & Cov(X,Z) \\ Cov(X,Y) & Var(Y) & Cov(Y,Z) \\ Cov(X,Z) & Cov(Y,Z) & Var(Z) \end{bmatrix}$$

$$R^T = \begin{bmatrix} -\sin B_0 \cos L_0 & -\sin B_0 \sin L_0 & \cos B_0 \\ -\sin L_0 & \cos L_0 & 0 \\ \cos B_0 \cos L_0 & \cos B_0 \sin L_0 & \sin B_0 \end{bmatrix} \quad (31)$$

Step 2: Adjustment of the network combining GNSS measurements ( $\Delta X$ ,  $\Delta Y$ ,  $\Delta Z$ ) and terrestrial distance measurements  $S_k$  ( $k=1, 2, \dots, n_2$ ).

In this step, it is necessary to set up observation equations for GNSS edge vectors and terrestrial edges, including the coefficient matrix of each vector and free term  $L_i$ :

$$V_i = A_i X + L_i \quad (32)$$

Each edge vector of  $\Delta x$ ,  $\Delta y$ ,  $\Delta z$  generates 3 observation equations as follows:

$$\left. \begin{aligned} v_{\Delta xy} &= -dx_i + dy_j + (x_j^0 - x_i^0 - \Delta x_{ij}) \\ v_{\Delta yj} &= -dy_i + dy_j + (y_j^0 - y_i^0 - \Delta y_{ij}) \\ v_{\Delta zj} &= -dz_i + dz_j + (z_j^0 - z_i^0 - \Delta z_{ij}) \end{aligned} \right\} \quad (33)$$

where,  $v_{\Delta x}$ ,  $v_{\Delta y}$ ,  $v_{\Delta z}$ : corrections of coordinate increments in the geographic system;  $dx$ ,  $dy$ ,  $dz$ : corrections of geographical coordinates for the point to be determined;  $x^0$ ,  $y^0$ ,  $z^0$ : approximate coordinates

$$\left. \begin{aligned} \text{where: } l_{\Delta x_{ij}} &= (X_j^0 - X_i^0) - \Delta x_{ij} \\ l_{\Delta y_{ij}} &= (Y_j^0 - Y_i^0) - \Delta y_{ij} \\ l_{\Delta z_{ij}} &= (Z_j^0 - Z_i^0) - \Delta z_{ij} \end{aligned} \right\} \quad (34)$$

$P_i$  matrix of the edge vector is calculated as follows:  $P_i = M_i^{-1}$

After that, the correction equations of measurements are generated. If it is an inclined edge, the corrected numerical equation takes the form as equation (35):

$$v_s = -\frac{(x_i^0 - x_j^0)}{S^0} dx_i - \frac{(y_i^0 - y_j^0)}{S^0} dy_i - \frac{(z_i^0 - z_j^0)}{S^0} dz_i + \frac{(x_i^0 - x_j^0)}{S^0} dx_i + \frac{(y_i^0 - y_j^0)}{S^0} dy_i + \frac{(z_i^0 - z_j^0)}{S^0} dz_i + l_s \quad (35)$$

If it is the horizontal edge, the correction equation is as follows:

$$v_{\kappa_i} = -\frac{(x_i^0 - x_j^0)}{D_{\kappa_i}} dx_i - \frac{(y_i^0 - y_j^0)}{D_{\kappa_i}} dy_i + \frac{(x_i^0 - x_j^0)}{D_{\kappa_i}} dx_i + \frac{(y_i^0 - y_j^0)}{D_{\kappa_i}} dy_i + l_{\kappa_i} \quad (36)$$

where  $l_{\kappa_i} = \sqrt{(x_i^0 - x_j^0)^2 + (y_i^0 - y_j^0)^2} - S_{\kappa_i}$ ;  $P_{S_i} = 1/m_{S_i}^2$

The general standard equation system is calculated according to the formula:

$$A^T P A = \sum_{i=1}^n a_i^T P_i a_i + \sum_{k=1}^{n_2} (a_k^S)^T P_k a_k^S \quad (37)$$

$$A^T P L = \sum_{i=1}^n a_i^T P_i l_i + \sum_{k=1}^{n_2} (a_k^S)^T P_k l_k^S \quad (38)$$

In which  $a_i^T$  is the coefficient of the correction equation of GNSS measurement value  $a_k^S$ ; is the coefficient system of correction equation of terrestrial edge measurements.

The general standard equation system is solved, an unknown vector  $X$  is gained as follows.

$$X = -(A^T P A)^{-1} (A^T P L) \quad (39)$$

$$V = A X + L \quad (40)$$

Step 3: Assessing the quality of the measurements after adjustment

For accuracy assessment, root mean square error of weight unit  $\sigma$  is computed as follows:

$$\hat{\sigma} = \sqrt{\frac{V^T P V}{3n + n_1 + n_2 - 3m}} \quad (41)$$

$$V^T P V = \sum_{i=1}^n v_i^T P_i v_i + \sum_{k=1}^{n_2} (v_k^S)^T P_k v_k^S \quad (42)$$

where  $n$  is the number of GNSS edges;  $n_1$  is the number of horizontal angles;  $n_2$  is the number of terrestrial edges;  $m$  is the number of unknown points.

Additionally, statistical standard  $\chi^2$  is used to assess the reliability of the adjustment results according to the distribu-

tion, if it meets the standard of  $\chi^2(\chi_{r,1-\alpha/2}^2 \leq VTPV \leq \chi_{r,\alpha/2}^2)$  then adjustment results are accepted, if  $\chi^2$  condition is not satisfied, the next step will be applied.

Step 4: If the observations contain coarse errors, the robust method with an empirical weight function computed by equation (26) is used to detect measurements that contain coarse errors.

## 4. Numerical example

### 4.1 Study area and materials

Lang Son province located at North East of Vietnam is a mountainous area, where there are many limestone quarries supplying raw materials to a large cement plant in Lang Son. The annual volume of limestone mining is very large; therefore, in order to serve the management of mining reserves, mapping the exploitation, setting up a precise geodetic network is essential. However, the topographical conditions of Lang Son quarry are complex (Figure 1), the placement of GNSS device at some control point is not satisfied due to unsatisfied threshold angle or near unsatisfactory power station. Therefore, to facilitate mine surveying, a 3D terrestrial - GNSS network was established. The combined network was illustrated in Figure 2. The network consists of 5 points, two original points are I, II, and three unknown points are III, IV, V. Due to the topographic conditions, GNSS device could not be placed at point I. Therefore, the edges I-III, I-IV, I-V were measured by an electronic total station (TS) with accuracy  $\sigma_{Si} = (2\text{mm} + 2\text{ppm} \cdot \text{Si})$  and edges II-III, II-IV, II-V, V-IV, II-IV were measured by GNSS technology with accuracy  $\sigma_{Si} = (3\text{mm} + 1\text{ppm} \cdot \text{Si})$ . The data of TS edge measurement are presented in Table 1, the data of GNSS measurement in the geocentric coordinate system are shown in Table 2.

### 4.2 Experimental framework

The experiment includes five steps illustrated in below Figure 3. Where,

Step #1. Reading GNSS and terrestrial data.

Step #2. Calculating and transforming measurement values of  $\Delta X$   $\Delta Y$   $\Delta Z$  and the covariance matrix from the geocentric coordinate system to the geographic coordinate system.

Step #3. Adjusting 3D terrestrial - GNSS geodetic network in case of the observations do not contain rough errors using the least-squares method. The adjustment results (i.e. correction  $v_i$ ) are considered as reference data for comparing to results of other Robust methods when the measurements contain rough errors.

Step #4. Assigning coarse errors:

Firstly, coarse errors with the corresponding magnitude of  $4\sigma$ ,  $5\sigma$ ,  $6\sigma$ , ...,  $9\sigma$ ,  $10\sigma$  ( $\sigma = 3\text{mm}$ ) are randomly assigned for measurement of . Then the deviation of the corrections is calculated as  $\Delta v_i = |v_i - v_i'|$  (39)

where  $v_i$  is the correction in case the measurements do not contain coarse errors;  $v_i'$  is the correction in case the measurements contain coarse errors.

Secondly, coarse errors with the corresponding magnitude of  $4\sigma$ ,  $5\sigma$ ,  $6\sigma$ , ...,  $9\sigma$ ,  $10\sigma$  ( $\sigma = 3\text{mm}$ ) are assigned randomly for other measurements.

Step #5. Coarse errors detection: The possibility of detecting coarse errors of experimental weighted function methods, L1-norm method, Tukey method, Danish method and Huber

method is investigated. The conditions to detect observations containing coarse errors are presented in equation (26).

#### 4.3 Results and discussion

In order to simplify the experiment, a computer module was programmed using Microsoft Visual C language. This module can read observation data, both of GNSS and terrestrial data; estimate and transform variance and co-variance matrix, and adjust integrated network using LSM or Robust estimation method if the network considered as containing outliers. The main interface of this module is illustrated in Figure 4.

After adjusting the combined network without coarse errors using LSM, the coarse errors with different value were added into the measured value of  $\Delta X_{v-iv}$ . Then the robust estimation using the five methods was performed. The corresponding deviation of the obtained correction of  $\Delta X_{v-iv}$  comparing to the case without coarse errors was computed by equation (39) and shown in Tab. 3. Additionally, the deviations of  $\Delta v$  to the assigned coarse error were shown in Tab. 4.

It is clear that when applying robust estimation using experimental weighting function, the deviation of  $\Delta v$  to the assigned coarse error is smallest, and it gradually decreases when the assigned coarse error increases. Especially, it is closed to 0 when the assigned coarse error reaches  $9\sigma$  (i.e. coarse error =  $9 \times 3\text{mm} = 27\text{mm}$ ). It means that identifying coarse errors based on robust estimation using experimental weighting function is easier than the other methods.

In the second case, the assumption that 2 measurements are containing coarse errors, they are and , and the assigned coarse error value is 15mm. The corrections obtained by the five robust estimation methods were shown in Figure 5.

It can be seen in Figure 5 that experimental weighting method detected coarse error value approximates the assigned

coarse error value (i.e. 15mm). This shows that, when using the experimental weighting function, the coarse error mainly affects the measurement value containing the coarse error and just slightly affects other measurements. Therefore, it is easy to identify the measurement containing coarse errors.

To assess the possibility of coarse errors detection by five methods, the percentage of observation containing coarse error detected was computed and Table 5. As can be seen in Table 5, the experimental weighting function method gives the best results compared to the L1-norm, Tukey, Danish and Huber methods, particularly when the measurement value contains errors less than 10% of the total number of measurements, the algorithm can detect 81–98%. The L1-norm, Tukey, Danish and Huber methods give lower results within a range from 50% to 86%. Therefore, when identifying rough errors for a 3D combined network, it is required to consider the correlation of GNSS measurements.

#### 5. Conclusions

The experimental result showed that the Robust estimation method applying an empirical weighting function could quickly and accurately detect measurements containing coarse errors. Processing data of 3D terrestrial-GNSS network in mine area using experimental robust method can eliminate measurements that contain rough errors; as a result, it improves the accuracy of mine surveying such as determining mining reserves, monitoring mine slide. Therefore, it can be an effective solution for the processing of the integrated geodetic network data for mine surveying in Vietnam.

Establishing 3D terrestrial-GNSS network of Lang Son mine ease the network positioning in the national reference system; as a result, exploitation planning and management is more convenient. It is useful to establish 3D terrestrial- GNSS network.

## Literatura – References

1. Box, G.E.P. (1953). "Non-normality and tests on variances." *Biometrika* 40: 318-355.
2. Cui Xizhang, Wu Zongchou, Tao Benzao, Liu Dajie, Yu Zhenglin, Sun Haiyan, Wang Xinzhou (2009). "Extend surveying adjustment." Publishing house of Surveying and Mapping, Beijing, China
3. Groten, E. (1977). "Combination of geometrical satellite with terrestrial data. In (Eds. Halmos F. and Samogyi J.): Optimization of design and computation of control networks." International Symposium in Sopron, 4–10 July, 1977). Akademiai Kiado, Budapest, 95–104.
4. Hampel F.R., Ronchetti E.M., Rousseeuw P.J. and Stahel W.A. (1986). "Robust Statistics, the Approach based on the influence functions." Wiley and Sons. New York.
5. Harvey, B.R. (1993). Survey network adjustments by the L1 method. *Australian Journal of Geodesy, Photogrammetry and Surveying* 59: 39-52
6. Huber, P.J., (1964). "Robust estimation of a location parameter." *Annals of Mathematical Statistics* 35: 73-101.
7. Huber, P.J., (1981). "Robust Statistics." John Wiley & Sons, New York.
8. Koch, K. R. (2014). "Robust estimations for the nonlinear Gauss Helmert model by the expectation maximization algorithm." *Journal of Geodesy* 88(3): 263-271.
9. Koch K.R. (2013). "Robust estimation by expectation maximization algorithm." *J Geod* 87(2):107-116.
10. Koch K.R., Kargoll B. (2013). "Expectation maximization algorithm for the variance-inflation model by applying the t-distribution." *J Appl Geod* 7:217-22
11. Krakiwsky, E.,J. and Thomson, D.B. (1974). Mathematical models for the combination of terrestrial and satellite Networks. *The Canadian Surveyor*, 28(5).
12. Kraup, T., and K. Kubik (1983). "The Danish Method; Experience and Philosophy." DGK, Reihe A.Heft 8: 131-134.
13. Thomson, D.B. (1976). Combination of Geodetic Networks. Technical Report No. 30, April 1976, Department of Surveying Engineering University of New Brunswick, Canada
14. Tukey I.W. (1962). "The future of data analysis." *Ann. Math. Statist.* 33: 1-67.
15. Yang, Y. (1999). "Robust estimation of geodetic datum transformation." *Journal of geodesy* 73: 268-274.
16. Wang Xinhou, Tao Benzao, QiuWeining, Yao Yibin (2006). "Advanced surveying adjustment." Publishing house of Surveying and Mapping, Beijing, China.
17. Yang Y. (1994). "Robust estimation for dependent observations." *Mans. Geod.* 19: 10-17.
18. Yang Y, He H and Xu G (2001). "Adaptively robust filtering for kinematic geodetic positioning." *Journal of Geodesy*, 75(2/3): 109-116.
19. Yang Y, Lijie Song, Tianhe Xu (2002). "Robust estimation for the Adjustment of Correlated GPS Network." *Journal of Geodesy, China*.

## *Zastosowanie metod estymacji odpornej w sieciach geodezyjnych zintegrowanych z 3D obserwacjami GNSS – studium przypadku w kamieniołomie Lang Son*

Niezawodność osnowy geodezyjnej wynika z niezawodności układu obserwacyjnego, który tę osnowę wyznacza oraz wyraża możliwość jego kontroli na wypadek zaistnienia błędów grubych, szczególnie w przypadku dużych osnów składających się z tradycyjnych sieci zintegrowanych z GNSS obserwacjami. Z drugiej strony, ponieważ tereny kopalni odkrywkowych w Wietnamie są często o silnie zróżnicowanej morfologii utworzone w wyniku eksploatacji górniczej a sieci kontrolne na obszarach górniczych powstawały w różnych okresach o różnych figurach geometrycznych i różnych metodach pomiarowych, więc jest zintegrowanie klasycznych sieci z GNSS obserwacjami zostało koniecznym zagadnieniem. W artykule, przedstawiono wyniki badania nad zastosowaniem metod estymacji odpornej w sieciach geodezyjnych zintegrowanych z 3D obserwacjami GNSS w kamieniołomie Lang Son. Eksperyment dotyczący przetwarzania połączonej sieci w kamieniołomie wapienia Lang Son (Wietnamie) wykazał, że proponowana metoda jest odpowiednim rozwiązaniem do przetwarzania tradycyjnej naziemnej sieci z 3D GNSS-owskimi obserwacjami w osnowie geodezyjnej służącej skutecznie do prac geodezyjnych w kamieniołomie Lang Son a w innych kopalniach odkrywkowych generalnie mówiąc.

**Słowa kluczowe:** estymacja odporna, sieć geodezyjna, dopasowanie sieci, estymacja metodą najmniejszych kwadratów, wartości odstające