Two-step Homogenization of Poroelastic Properties of a Limestone

Trieu HUNG TRUONG1), Nguyen NGOC BIEN1), Pham DUC THO1), Vu MINH NGOC3), Do NGOC ANH1) Nguyen SY TUAN3), Tran NAM HUNG4), Nguyen THI THU NGA4)

1) Hanoi University of Mining and Geology, Hanoi, Vietnam, Vietnam; email: trieuhungtruong@humg.edu.vn; nguyennngobien@humg.edu.vn; phamductho@humg.edu.vn; dongoanh13@humg.edu.vn
2) Institute for Computational Science, Ton Duc Thang University, Ho Chi Minh City, Vietnam; email: vuminhngoc@htdu.edu.vn
3) Institute of Research and Development, Duong Tan University, Da Nang, Vietnam; email: stuan.nguyen@gmail.com
4) Le Quy Don Technical University, Institute of Techniques for Special Engineering, Hanoi, Vietnam; email: nguyennnga@lqdtu.edu.vn

http://doi.org/10.29227/IM-2020-02-06
Submission date: 06-03-2020 | Review date: 22-09-2020

Abstract

This study aims at deriving the effective poroelastic properties of the oolitic limestones based on the Hashin composite sphere assemblage (CSA) micromechanical theory. The microstructure of oolitic limestones generally exhibits an assemblage of grains (oolites) surrounded by a matrix. Grain and matrix are linked via the interfacial transition zone (ITZ). Pores exist in these three material phases (oolite, ITZ and matrix). A two-step homogenization method is proposed. The first step consists of upscaling the properties of each porous phase (i.e. porous oolite, porous ITZ and porous matrix) in which each phase contains two sub-phases including pore and solid. The differential self-consistent scheme is used for the first step. At the second step, the three different porous constituents (oolite, ITZ and matrix) are assembled in a CSA model. A mathematical analogy between thermoelasticity and poroelasticity is used to obtain the effective poroelastic properties. A comparison between the proposed model and test data on the oolitic limestone from Bourgogne (France) helps to calibrate the model parameters and to highlight the role of ITZ phase.

Keywords: oolitic limestone; tow-step homogenization; CSA; poro-elastic properties; differential self-consistent

1. Introduction

Knowledge of the poroelastic properties (drained and undrained bulk and shear moduli; Biot coefficient; Biot modulus; Skempton’s coefficient, etc.) is significant for numerous environmental and industrial engineering problem. Indeed, thermo-poroelastic approach is usually considered for the design of geological sequestration of CO2 (2009). Therefore, determination and prediction of physical properties of carbonate rocks are paramount in geophysics and geomechanics. Field measurement (e.g. log well technique) allows measuring the elastic properties (compressional and shear ultrasonic wave velocities) of the host rock (Nguyen et al., 2015, 2017). However, laboratory test on the sample, including drained, undrained, pore pressure loading and unjacketed tests, must be had recourse to determine the complete poroelastic properties (Lion et al., 2004). Some properties can be measured directly or indirectly from laboratory test (Lion et al., 2004). Besides, theoretical and numerical methods such as micromechanical approach (Dormieux et al., 2006; Giraud et al., 2012; Levin & Alvarez-Tostado, 2006; Nguyen et al., 2017) or machine learning (Madhubabu et al., 2016) have been widely used to predict and extrapolate the poroelastic properties of carbonate rock from measured data.

This study aims at proposing a theoretical model based on two-steps homogenization of poroelastic properties of a limestone from Bourgogne (France) (Lion et al., 2004). This limestone belongs to the oolitic rocks. Microstructure observation evidence that the limestone is a formation of concentric ooids (oolitic grain) nearly spherical coated by a thin layer with high porosity and then a matrix. The oolite consists of micrite and pore, while the surrounding matrix is constituted by cementitious sparite and pore. Many studies in the literature on various limestones have shown similar microstructure (Chen et al., 2017; Fabre & Gustkiewicz, 1997; Grigic, D, 2011; Hart & Wang, 1995). The composite sphere assemblage (CSA) models proposed by Hashin (Hashin, 1962) is quite suitable for this microstructure of limestone. Moreover, CSA model is also appropriated for materials containing intermediate and large volume fractions of grains when estimating their poroelastic properties. Whereas, the matrix-inclusion meth-
The derivation of the poroelastic properties of the oolitic limestone, which is considered as a concentric spherical assemblage of three phases: ooids, interphase (ITZ) and cementitious matrix. Each phase is constituted by pore and calcite mineral (micrite for ooids and ITZ and sparite for matrix). In the following, pore and calcite (micrite, sparite) are referred to the microscale. Ooid, ITZ and cementation matrix are designated the mesoscale. A representative elementary volume (REV) of limestone is envisaged as a macroscale. The first upscaling from the microscopic to the mesoscopic scale (step I) is performed based on the differential self-consistent scheme. The second upscaling from the mesoscopic to macroscopic scale, which is carried out by the CSA four-phase model (Hashin & Monteiro, 2002). Fig. 1 shows the schematically principle of the two-step upscaling to derive the poroelastic properties of the limestone.

We consider a REV Ω of limestone constituted by three phases: ooids (o), interphase (i) and matrix (m). At the microscopic scale, the skeleton solid of three constituents (o, i, m) is assumed to be spherical while ITZ and matrix are represented by spherical shells. The case without ITZ is also considered to show the role of ITZ when deriving the poroelastic properties of the oolitic rock. Three-phase CSA model in the latter case is the well-known generalized self-consistent scheme. The derivation of the poroelastic properties of porous rock is based on the correspondence between the linear poroelasticity and the linear thermoelasticity (Berryman, 1997; Berryman & Milton, 1991, 1992; Levin & Alvarez-Tostado, 2003). Properties of each constituent at the first step of homogenization are taken from the literature. Comparison with the measurement of the limestone from Bourgogne (France) shows the accuracy of the proposed homogenization-based model.

2. A two-step homogenization model

Two-step homogenization model (micro-meso and meso-macro) is proposed to derive the overall poroelastic properties of the oolitic rock. Each phase is constituted by pore and calcite mineral (micrite for ooids and ITZ and sparite for matrix). In the following, pore and calcite (micrite, sparite) are referred to the microscale. Ooid, ITZ and cementation matrix are designated the mesoscale. A representative elementary volume (REV) of limestone is envisaged as a macroscale. The first upscaling from the microscopic to the mesoscopic scale (step I) is performed based on the differential self-consistent scheme (Norris, 1985; Zimmerman, 1996). The second upscaling from the mesoscopic to macroscopic scale, which is carried out by the CSA four-phase model (Hashin & Monteiro, 2002). Fig. 1 shows the schematically principle of the two-step upscaling to derive the poroelastic properties of the limestone.
Micro-meso transition (step I)

The first step of micro-macro upscaling results in the overall poroelastic properties of three constituents including porous oolite; porous matrix and porous interphase. Each constituent is composed of pore and skeleton solid. As shown by (Nguyen et al., 2011), differential self-consistent scheme seems to be the most appropriated for two-step homogenization models for poroelasticity, which is thus adopted in this study.

Effective linear poroelastic constitutive equations at the mesoscale, with the assumption of zero initial stress and zero initial pore pressure, are written as follows (Dormieux et al., 2006).

\[
\Sigma = C I : \varepsilon - B \phi \frac{P}{N} \tag{6}
\]

\[
\phi \varepsilon = B I : \phi \varepsilon + \frac{P}{N} \tag{7}
\]

where \( \Sigma, E \) represents the mesoscopic stress and strain tensor; \( \phi \) the porosity; \( P \) the pore pressure; \( B \) the Biot tensor coefficient and \( N \), the solid Biot modulus (\( j = o, i, m \)).

Micro-macro compatibility condition yields:

\[
\bar{B}^M_j = \frac{1}{2} \left[ (I - \sigma_j : C_j) \right], \quad \frac{1}{N_j} = (\bar{B}^j_\phi : \bar{B}^j_\phi) : \frac{\bar{B}^M_j}{N_j} \tag{8}
\]

in which, \( \sigma \) is the second-order identity tensor.

For the particular case where both solid constituent \( j \) and solid skeleton are isotropic and homogeneous, we have

\[
\bar{B}^M_j = \frac{1}{2} \left[ (I - \sigma_j : C_j) \right], \quad \frac{1}{N_j} = (\bar{B}^j_\phi : \bar{B}^j_\phi) : \frac{\bar{B}^M_j}{N_j} \tag{9}
\]

Assuming the solid phase and pore take the spherical shape at the microscopic scale, differential self-consistent approach gives an estimation of elastic properties such as (Norris, 1985; Zimmerman, 1996)

\[
\bar{E}^M_j \varphi \phi - \bar{E}^j_\phi \phi - \bar{E}^j_\phi \phi - \bar{E}^M_j \phi \phi - \frac{P}{N} \tag{10}
\]

where \( \bar{E}^M_j \varphi \), \( \bar{E}^j_\phi \phi \), \( \bar{E}^j_\phi \phi \), \( \bar{E}^M_j \phi \phi \), \( \frac{P}{N} \) are the Biot tensor coefficient and \( N \), the solid Biot modulus (\( j = o, i, m \)).
in which,

\[
\kappa_j = \frac{1 - 3v_j}{2(1 + v_j)} \frac{2\kappa_j - 4\mu_j}{6\kappa_j} \tag{11}
\]

Therefore, the mesoscopic bulk and shear moduli are the root of the nonlinear equations 10. The Biot coefficient and Biot modulus are determined from the equation 8.

\[
\begin{align*}
\beta_j &= \frac{\kappa_j}{3\kappa_j + 4\mu_j} & \sigma_j &= \frac{2(\kappa_j + 2\mu_j)}{3(3\kappa_j + 4\mu_j)} \tag{12}
\end{align*}
\]

Once the mesoscopic poroelastic properties are formed by equations 10, 11, and 12, they are introduced in the next step (step II) to determine the overall properties of the limestone based on the CSA model (four-phase model).

Meso-macro transition (step II)

The second step (step II) consists in the upscaling from the mesoscale (i.e. oolite, ITZ and matrix) to the macroscale (limestone) by using the three-phase composite sphere model (or four-phase model). The homogenized linear poroelastic constitutive equation reads

\[
\begin{align*}
\Sigma &= C_{\text{hom}}^\text{II} \cdot \varepsilon + B_{\text{hom}}^\text{II} \cdot \sigma \\
\phi \cdot \Phi &= B_{\text{hom}}^\text{II} \cdot \phi = \frac{\varepsilon}{N_j} - \frac{\sigma}{N_j} \tag{13}
\end{align*}
\]

where \(C_{\text{hom}}^\text{II} \) and \(B_{\text{hom}}^\text{II} \) denote the fourth-order effective macroscopic stiffness tensor; the second-order Biot coefficient tensor and the solid Biot modulus.

It is worth to notice that the constitutive equations of linear poroelasticity and those of linear thermoelasticity are similar (Berryman, 1997; Berryman & Milton, 1991, 1992; Levin & Alvarez-Tostado, 2003). Therefore, the poroelastic properties of three-phase CSA (or four-phase model) can be obtained from the known results of such a model in the framework of the thermoelastic theory (He & Benveniste, 2004; Herve, 2002; Herve & Zaoui, 1993). Hereafter, the final solution of poroelastic properties of limestone in step II is shown. Detailed development resulting in these results were presented in (Nguyen, 2010).

The bulk modulus:

\[
\kappa_{\text{hom}}^\text{II} = \frac{3\kappa_k + 4\mu_k}{3 - \frac{f_0^s \Phi_0^s + f_0^b \Phi_0^b}{f_0^s \Phi_0^s + f_0^b \Phi_0^b}} \tag{14}
\]

where:

\[
\begin{align*}
\delta_0 &= \frac{3\kappa_k + 4\mu_k}{3\kappa_k + 4\mu_k} & \delta_1 &= \frac{3\kappa_k + 4\mu_k}{3\kappa_k + 4\mu_k} \\
\delta_2 &= \frac{k_k - k_m}{3k_k + 4\mu_k} & \delta_3 &= \frac{k_k - k_m}{3k_k + 4\mu_k} \tag{15}
\end{align*}
\]

and the Biot coefficient:

\[
\beta_{\text{hom}}^\text{II} = \frac{f_0^s \Phi_0^s + f_0^b \Phi_0^b}{f_0^s \Phi_0^s + f_0^b \Phi_0^b} \tag{16}
\]

With:

\[
\begin{align*}
\gamma_0 &= \frac{f_0^s \Phi_0^s + f_0^b \Phi_0^b}{f_0^s \Phi_0^s + f_0^b \Phi_0^b} & \gamma_1 &= \frac{f_0^s \Phi_0^s + f_0^b \Phi_0^b}{f_0^s \Phi_0^s + f_0^b \Phi_0^b} \\
\gamma_2 &= f_0^s \kappa_k + f_0^b \kappa_m & \gamma_3 &= f_0^s \kappa_k + f_0^b \kappa_m \tag{17}
\end{align*}
\]

and the Biot modulus:

\[
1 - \frac{1}{N_j} \sum_{N_j} \frac{1}{N_j} \frac{N_j}{N_j} \tag{18}
\]
Fig. 6. Variation of Biot coefficient of ooid and cement matrix versus their porosity (step I)

In which:

\[ N_{\text{mm}} = -\frac{3k_{w}}{k_{1}} \left[ 3k_{w} + 4\mu_{e} \right] \left( k_{1} - k_{w} \right) f_{c} \]

\[ k_{w} = \frac{3k_{w} + 4\mu_{e} \left( k_{1} - k_{w} \right) f_{c}}{4\mu_{e} + 3k_{w} f_{c}} \]

\[ 1 = \frac{f_{c}}{N_{\text{mm}}} + \frac{1 - f_{c}}{N_{w}} + \frac{3\left( k_{1} - k_{w} \right)^{2} f_{c} \left( 1 - f_{c} \right) f_{s}}{3k_{w} \left( k_{1} - k_{w} \right) + 3k_{i} + 4\mu_{e}} \]

The effective Biot modulus obtained by the following relation:

\[ \frac{1}{M_{\text{mm}}} = \frac{1}{N_{\text{mm}}} + \frac{f_{c}}{k_{1}} \]

Hashin (Hashin, 1962) considered only two layers and obtained an identical formulation for bulk modulus as the solutions …..

3.1. Studied oolitic rock

The material studied in this work is an oolitic limestone extracted from Bierry-les-Belles-Fontaines, in Bourgogne (France). Different observation techniques including X-ray diffraction (XRD), optical microscopy (OM) and scanning electron microscopy coupled with an energy dispersive spectrometer (SEM-EDS) were used for petrographic study. Microstructure characterization evidenced that the limestone is an assemblage of ooids of size 100–1000 µm made of concentric spheres of calcite (Fig. 2). Sparite (coarse crystalline calcite) and minor micrite (micro-crystalline calcite with a grain size finer than 4 µm) are present in the cement matrix. SEM photomicrograph shows that an ooid is constituted by micrite crystals and microporosity (Fig. 3). At a centimetric scale, the rock can be considered as homogeneous. Both sparite and micrite are made of 100% CaCO₃.

Poro-elastic properties of the considered rock were characterized by an experimental device enabling confining pressure (≤ 60 MPa) to be applied, injected or expelled fluid mass and strains to be recorded. The main poroelastic properties can be determined either by direct or indirect measurement. Drained bulk modulus \( k_{d} \) is measured by keeping a constant pore pressure (drained condition \( \Delta P_{f} = 0 \)) when increasing the isotropic confining pressure. Thus, the drained bulk modulus is the ratio between the variation in confining pressure \( \Delta P_{c} \) and the variation in volumetric strain \( \Delta v \). Undrained bulk modulus \( k_{u} \) and Skempton's coefficient \( B \) are measured when increasing the confining pressure on the saturated sample under undrained condition. \( B \) is the ratio between the variation in pore pressure \( \Delta P_{p} \) and \( \Delta P_{c} \), \( k_{i} \) is the ratio between \( \Delta P_{i} \) and \( \Delta v \). The matrix bulk modulus \( k_{s} \) is determined when controlling \( \Delta P_{f} = \Delta P_{c} \) and \( k_{s} \) is the ratio between \( \Delta P_{f} \) and \( \Delta v \).
and porosity $\phi$ are known, the Biot coefficient $b$ and the Biot modulus $M$ can be determined indirectly from equations 9 and 23. Besides, these two poroelastic parameters can also be measured directly by the experimental device. To measure Biot coefficient $b$, a fluid injection into the sample under the drained condition when keeping a constant confining stress ($\Delta P_c = 0$) was performed. We obtain $b = k \times \Delta \varepsilon_v / \Delta P_f$. On the contrary, $M$ can be measured from an undrained hydrostatic test. A recording of pore pressure variation yields $M = -(1/b) \times \Delta P_f / \Delta \varepsilon_v$. Details about the measurement and test data were presented by (Lion et al., 2004).

3.2. Comparison between the proposed model and test data

The parameters of the two-step homogenization models include the bulk and shear modulus of micrite and sparite ($k_{mi}, \mu_{mi}, k_{sp}, \mu_{sp}$); porosity of three phases $f_{po}, f_{pi}, f_{pm}$ and the volume fractions of oolites. The elastic properties of micrite and sparite were deduced from microindentation tests on Lavoux limestone (France) (Giraud et al., 2012). Moreover, the repartition of porosity within that limestone at the mesoscale was also given by Giraud (Giraud et al., 2012). These known parameters from Lavoux limestone is adopted in this study for Bourgogne limestone (Table 1). Fig. 4 to Fig. 7 show the poroelastic properties of porous ooid and porous matrix as a function of their porosity, which are resulted from the step I. The results of step II of the proposed model are given in Table 2 for two cases with and without ITZ. Comparison of model results to the test data of (Lion et al., 2004) shows that the four-phase model is quite appropriated to predict the poroelastic properties of oolitic limestone. Moreover, this comparison also evidences the role of ITZ in the prediction.

4. Conclusion

This paper presents an analytical model to predict the poroelastic of oolitic rock. The studied material appears as an assemblage of three layers including spherical oolitic grain coated by double spherical shells mince interphase and cement matrix. Ooid and interphase are made by micrite (micro-crystalline calcite) and micropores, while cement matrix is constituted by sparite (coarse crystalline calcite) and micropores. Based on the petrographic characterization of this rock, a two-step homogenization model is proposed. The step I consists of the micro-meso upscaling, which homogenize the properties of ooid, ITZ, and matrix based on the differential self-consistent scheme. The step II adopts the three phase composite sphere model (four-phase model) to obtain the overall poroelastic of the rock. The derivation process is based on the correspondence between constitutive equations of poroelasticity and those of thermoelasticity. The validation of the proposed model is shown by the comparison between test data. The comparison between three-phase model (without ITZ) and four-phase model shows the significant role of the interphase.

Acknowledgement

This work is financially supported by the Ministry of Education and Training of Vietnam, Grant No. B2019-MDA-562-18.
Literatura – References


Dwu-etapowa homogenizacja właściwości poroelastycznych wapienia

W artykule, przedstawiono wyniki badań efektywnych właściwości poroelastycznych wapieni oolitowych w oparciu o teorię mikro-mechaniczną złożonego zespołu kul Hashin (CSA). Mikrostruktura wapieni oolitycznych wykazuje generalnie zbiór ziaren (oolitów) otoczonych matrycą. Ziarno i matryca są połączone za pośrednictwem międzyfazowej strefy przejściowej (ITZ). W tych trzech fazach materiału (oolit, ITZ i matryca) istnieją pory. Zaproponowano dwuetapową metodę homogenizacji. Pierwszy etap polega na zwiększeniu skali właściwości każdej porowatej fazy (tj. Porowatego oolitu, porowatej ITZ i porowatej matrycy), w której każda faza zawiera dwie podfazy: porową i stałą. W pierwszym etapie zastosowano różnicowy schemat samouzgodnienia. Na drugim etapie trzy różne porowate składniki (oolit, ITZ i matryca) są składane w modelu CSA. Matematyczne analogie między termosprężystością a poroelastycznością są wykorzystywane do uzyskania efektywnych właściwości poroelastycznych. Porównanie proponowanego modelu z danymi testowymi dotyczącymi wapienia oolitycznego z Bourgogne (Francja) pomaga skalibrować parametry modelu i podkreślić rolę fazy ITZ.

Słowa kluczowe: wapienie oolityczne; homogenizacja dwuetapowa; CSA; właściwości poroelastyczne; zróżnicowana spójność własna.