Optimisation of the Costs of Rebates Using the Monte Carlo Method

Beata TRZASKUŚ-ŻAK[1], Andrzej ŻAK[2]

[1] (First Author) dr hab. inż.; AGH University of Science and Technology in Kraków, Faculty of Mining and Geoengineering, Department of Economics and Management in Industry, Mickiewicza 30, 30-059 Kraków, Poland; e-mail: t-zak@agh.edu.pl
[2] (Corresponding Author) dr hab.; AGH University of Science and Technology in Kraków, Faculty of Applied Mathematics, Department of Discrete Mathematics, Mickiewicza 30, 30-059 Kraków, Poland; e-mail: zakandrz@agh.edu.pl

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Abstract

In the paper, the use of Monte Carlo method to simulate the costs of rebates allowed by the analysed Polish open-pit rock mineral resources mine was proposed. Allowing rebates to recipients of products or services is one of accounts receivable management methods.
In this paper, the Monte Carlo method is used to determine the expected value of allowed rebates in the following steps of analysis by using an appropriate algorithm.
The application of the method was presented on the data of Polish open-pit mine and its selected customers. The selection was based on high numbers of monthly transactions.

Keywords: receivables management, rebates, The Monte Carlo Method

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Introduction

The literature does not limit the process of accounts receivable management towards the clients to one pattern. The choice of an appropriate solution belongs to the seller, who should use the following: monitoring recipients, securing receivables, receivables turnover, trade credit and rebate (discount) (Kłak 2006; Trzaskuś-Żak, 2013).

For a specific company, the accounts receivable management process will depend not only on measurable factors, but also on the seller's inclination to take risks. It can be therefore stated that accounts receivable management, including credit policy, is executed in three ways (Czekaj and Dresler 1998; Panfil 2004; Sierpińska and Wędzki 1998):

– conservatively, when the seller conditions the provision of goods or services on prior payment, trade credit is given only to long-term and reliable counterparties or after introducing appropriate safety measures,
– moderately, without strict solutions described above, but still using systematic monitoring of the recipients and requiring payment before delivery,
– aggressively, giving trade credit to practically every recipient without taking risk into account. It is risky per se and used in cases of planned client base expansion or sale scale increase. Aggressive credit policy is recommended in three cases: when a new product is launched, when there is excess production capacity, or when competitive situation on the market requires it.

There are many research methods in accounts receivable management. In accounts receivable monitoring, index, scoring, logit, and discriminant methods are used (Kreczmańska-Gigol 2010; Sierpińska and Wędzki1998; Czekaj and Dresler 1998).

Accounts receivable management optimisation methods include incremental analysis, marginal analysis, expert method, linear programming, and binary linear programming (Kłak 2006; Sierpińska, Wędzki 1998; Trzaskuś-Żak, Czopek 2013; Trzaskuś-Żak, Żak 2013).

Accounts receivable management, including rebates, consists in fulfilling diverse goals of a company, mine company in this case,([Fuksa 2009, Fuksa 2010; Nowak 2009; Kłak 2006; Michalski 2010; Portalska and Kornatowicz 2003), such as:

– improvement or maintenance of financial liquidity,
– profit increase,
– increased use of spare mining-processing capacities,
– finding new customer base or expanding the existing one,
– ways of fighting competition.

In accounts receivable management and trade credit, rebate, commonly understood as a discount of a fixed price of goods or services, or price reduction (lowering price of goods or service usually in whole deals) is widely used. There are two types of rebates: allowed before fixing a price and delivering the goods and allowed after delivery.

The rebate is also defined in Article 29a.1–7 of the act on VAT tax, which includes the following statement:

Tab. 1. Expected values of the percentage recognition of paid-in-term receivables for individual months [Source: own elaboration]
Tab. 1. Wartości oczekiwane odsetka należności terminowych dla poszczególnych miesięcy

Month	Before rebates [%]	After rebates [%]
January	72,52	88,34
February	73,50	92,19
March	74,51	90,47
April	86,04	95,53
May	84,85	95,93
June	84,62	94,88
July	83,51	95,22
August	81,15	94,74
September	86,35	96,34
October	82,54	95,60
November	74,79	92,57
December	80,57	91,47

The taxable amount is everything what the supplier, or service provider has received or is to receive from the purchaser, recipient or a third person, including received grants, subsidies and other payments of a similar nature that have a direct impact on the price of the goods supplied or the services provided by the taxpayer.

The taxable amount does not include price reductions in the form of a rebate for early payment, and provided to the purchaser or recipient the price discounts and reductions that are included at the time of sale (VAT tax Act, 2018).

Thus, the act enumerates a finite list of terms as financial rebate variants, and their short interpretation is as follows:

– reduction means lowering price of goods or service price, usually in wholesale deals,
– discount is lowering the price for a purchaser paying with cash or before the payment date defined in the agreement.

Sellers also use a system of product prizes, the so-called product rebate allowed in order to acquire customers. The purchaser, in exchange for buying a certain amount of goods, is given an additional premium (additional unit(s) of ordered goods). Product rebate concerns the same goods, because otherwise it would be a bonus sale.

Rebates (price reductions, discounts) are an effective way of elimination of negative financial phenomena, i.e. non-collectable or overdue receivables. Rational accounts receivable management gives the company an opportunity to decide which types of price reduction should it offer to the recipients.

Application of the Monte Carlo method to account receivables management of the analysed open-pit rock mineral resources mine

The Monte Carlo method may be used, among other things, for (Kopczewska et al. 2009):

– The development of prediction models based on known input data and their probabilities.
– Integrating – geometrically determining the area under a curve.
– Probability calculation – when calculation algorithms are complicated.
– Finance – valuating financial instruments.
– Optimisation problems.

The Monte Carlo method simulates a real situation by generating random variables [Wodarski 2009]. It is an interesting tool for quantitative problem solving when analytic methods based on formulas, estimators, etc. fail. The simulation approach allows finding solutions of advanced quantitative problems numerically in risky and uncertain situations – when the achieved values are known, but their probabilities are not. Simulation consists in drawing many possible scenarios and inferring based on their empirical distribution. Simulation is a tool allowing imitating a real system and should be treated as a model-mechanism of an examined process. It allows testing the influence of variable input date on given results (Kopczewska et al. 2009).

In general, the Monte Carlo method has the following steps (Magda et al. 2002):

1) preparing a set of input data,
2) dividing the input data into deterministic data and random data,
3) statistical processing of random data,
4) computer simulation using Monte Carlo method,
5) statistical processing and analysis of the obtained results,
6) elaboration of conclusions and recommendations for practical use.

The paper proposes using the Monte Carlo method to simulate the costs of allowed rebates. It will be achieved by using an appropriate algorithm. In the first steps of the analysis, individual probability of using a rebate was assigned to each of the selected 50 recipients of the analyzed mine's product taken into account, expected value of the percentage of term receivables was assumed as 90%. In the case of 80.84% share of term receivables in all the receivables of selected 50 recipients of the analysed mine. The rebate was proposed to chosen recipients for earlier payment. In the following steps of the analysis, the Monte Carlo method was used and a certain number of simulations (100 in the analysed case) aimed at determining the expected value of the cost of the allowed rebates was made. Formulas (1) to (7) were used for calculating the expected value of the cost of the allowed rebates.

The selection was based on high numbers of monthly transactions. As a result of using the Monte Carlo method, the expected value of the costs of the rebates proposed by the mine, which was 5,388,813.41, was obtained.

The calculations, the results of which were presented in the paper, were made using available computer programmes, i.e. STATISTICA, MsExcel.

A typical task in Monte Carlo method is calculating the expected value

$$\theta = E_{\Pi} f(X) \tag{1}$$

where:
X – random variable with probability distribution (Π) on the area (X), whereas $f : X \to R$.

The Monte Carlo method consists in generating (n) independent random variables $X_1, X_2, \ldots, X_n$ of identical distribution (Π) and using the expected value ($\hat{\theta}_n$), i.e. the average of the sample, as an estimator.

In the paper, the random variable is the vector:

$$X = (x_1, x_2, \ldots, x_{110}) \tag{2}$$

composed of 110 independent zero-one random variables:

$$x_i = \begin{cases} 1 & \text{with probability of } p_j \\ 0 & \text{with probability of } 1 - p_j \end{cases} \tag{3}$$

The xi = 1 value refers to a situation in which the recipient (j) uses the rebate in the analysed month. The probability (p_j) of using a rebate by the recipient (j) is determined based on careful monitoring of the recipients. Due to the lack of such data, the following procedure was used in the paper. Initially, all probabilities were set to zero. After that, each of them was increased by a component proportional to the percentage of term receivables of a given recipient.

The aim was to reflect the intuition that the recipients that have no problem with timely payment will be more likely to use a rebate. The following formula was used:

$$p_j = 0,5 + \frac{q_j}{2} \tag{4}$$

where:
$0 \le q_j \le 1$ - percentage of recipient's term receivables (j) (during the whole year).

For each (X) vector, the cost of allowed rebates K(X) was calculated using formulas (5), (6), and (7).

The cost of trade credit was determined using the following formula:

$$K_k = \sum_{s=1}^{T} \frac{r}{100} \cdot \frac{t_s}{360} \cdot N_s \tag{5}$$

where:
r – interest rate of the existing bank loan; 9.5 %,
ts – period of receivables collection, days (e.g. t_1=30, t_2=45,…,t_{12} = 360),
N_s – value of receivables with term of payment ts.

Rebate costs were calculated using the following formula:

$$k_j = \Delta P_j - K_k^{(j)} \tag{6}$$

where:
ΔP_j – difference in income from recipient j,
$K_k^{(j)}$ – cost of trade credit allowed to the recipient j.

Total cost of allowed rebates was determined using the following formula:

$$K_{cR} = \sum_{j=1}^{n} x_j \cdot k_j \tag{7}$$

where:
K_{cR} – total cost of rebates allowed by the mine
k_j – cost of allowing a rebate to the j recipient,
$x_j \in \{0,1\}$
j = 1, …, n – particular recipients.

In order to determine the expected EK(X) – of the total cost of rebates, and the expected value ENt(X) – of term receivables for a given month, 100 vectors X were generated following the distributions described above.

Results

Simulation methods require conducting an appropriate number of experiments in order to achieve high

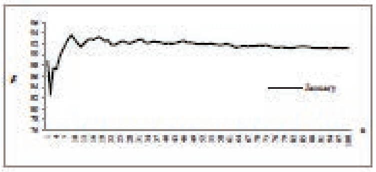

Fig. 1. Estimator ($\hat{\theta}_n$) for $EN_t(X)$ for January [*Source: own elaboration*]

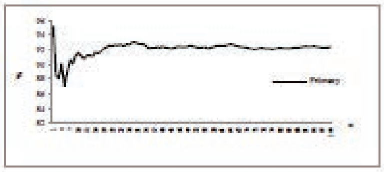

Fig. 2. Estimator ($\hat{\theta}_n$) for $EN_t(X)$ for February [*Source: own elaboration*]

Rys. 1. Kształtowanie się estymatora ($\hat{\theta}_n$) dla ENt(X) dla miesiąca stycznia. Źródło: opracowanie własne

Rys. 2. Kształtowanie się estymatora ($\hat{\theta}_n$) dla ENt(X) dla miesiąca lutego. Źródło: opracowanie własne

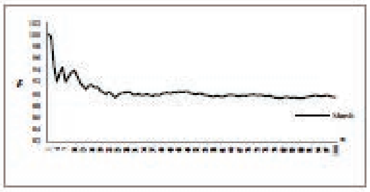

Fig. 3. Estimator ($\hat{\theta}_n$) for $EN_t(X)$ for March [*Source: own elaboration*]

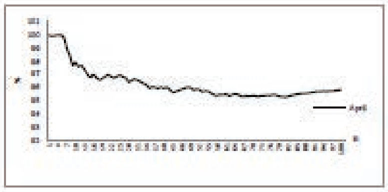

Fig. 4. Estimator ($\hat{\theta}_n$) for $EN_t(X)$ for April [*Source: own elaboration*]

Rys. 3. Kształtowanie się estymatora ($\hat{\theta}_n$) dla ENt(X) dla miesiąca marca. Źródło: opracowanie własne

Rys. 4. Kształtowanie się estymatora ($\hat{\theta}_n$) dla ENt(X) dla miesiąca kwietnia. Źródło: opracowanie własne

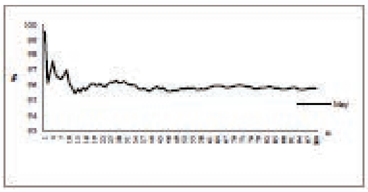

Fig. 5. Estimator ($\hat{\theta}_n$) for $EN_t(X)$ for May [*Source: own elaboration*]

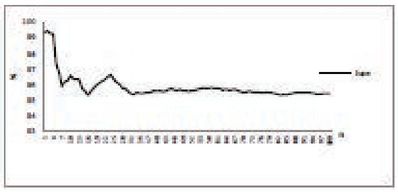

Fig. 6. Estimator ($\hat{\theta}_n$) for $EN_t(X)$ for June [*Source: own elaboration*]

Rys. 5. Kształtowanie się estymatora ($\hat{\theta}_n$) dla ENt(X) dla miesiąca maja. Źródło: opracowanie własne

Rys. 6. Kształtowanie się estymatora ($\hat{\theta}_n$) dla ENt(X) dla miesiąca czerwca. Źródło: opracowanie własne

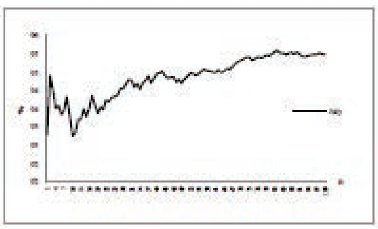

Fig. 7. Estimator ($\hat{\theta}_n$) for $EN_t(X)$ for July [*Source: own elaboration*]

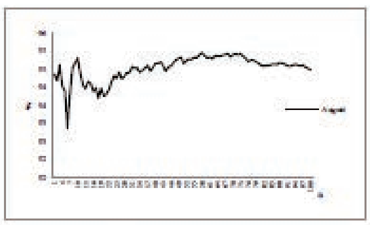

Fig. 8. Estimator ($\hat{\theta}_n$) for $EN_t(X)$ for August [*Source: own elaboration*]

Rys. 7. Kształtowanie się estymatora ($\hat{\theta}_n$) dla ENt(X) dla miesiąca lipca. Źródło: opracowanie własne

Rys. 8. Kształtowanie się estymatora ($\hat{\theta}_n$) dla ENt(X) dla miesiąca sierpnia. Źródło: opracowanie własne

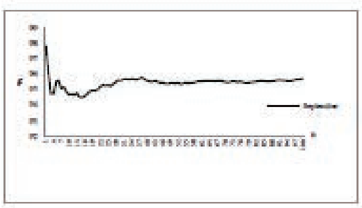

Fig. 9. Estimator ($\hat{\theta}_n$) for $EN_t(X)$ for September [*Source: own elaboration*]

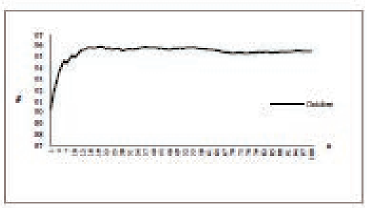

Fig. 10. Estimator ($\hat{\theta}_n$) for $EN_t(X)$ for October [*Source: own elaboration*]

Rys. 9. Kształtowanie się estymatora ($\hat{\theta}_n$) dla ENt(X) dla miesiąca września. Źródło: opracowanie własne

Rys. 10. Kształtowanie się estymatora ($\hat{\theta}_n$) dla ENt(X) dla miesiąca października. Źródło: opracowanie własne

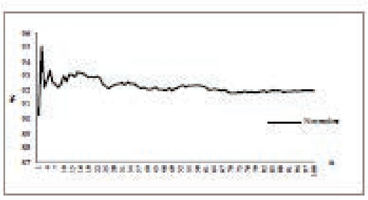

Fig. 11. Estimator ($\hat{\theta}_n$) for $EN_t(X)$ for November [*Source: own elaboration*]

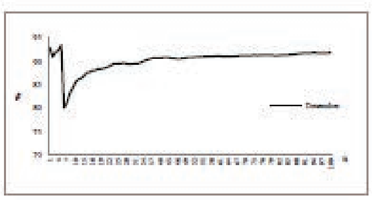

Fig. 12. Estimator ($\hat{\theta}_n$) for $EN_t(X)$ for December [*Source: own elaboration*]

Rys. 11. Kształtowanie się estymatora ($\hat{\theta}_n$) dla ENt(X) dla miesiąca listopada. Źródło: opracowanie własne

Rys. 12. Kształtowanie się estymatora ($\hat{\theta}_n$) dla ENt(X) dla miesiąca grudnia. Źródło: opracowanie własne

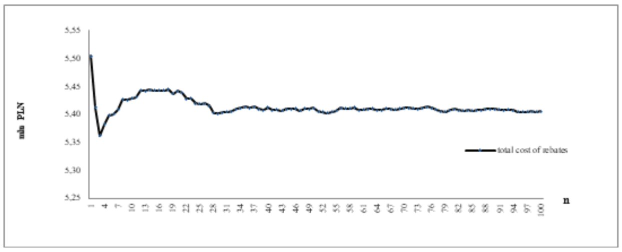

Fig. 13. Estimator ($\hat{\theta}_n$) for the expected value EK(X) of rebates cost in the whole year
Rys. 13. Kształtowanie się estymatora ($\hat{\theta}_n$) dla wartości oczekiwanej EK(X) kosztu rabatów dla całego roku. Źródło: opracowanie własne

accuracy in determining the examined value. The necessary number of experiments (n) depends of the desired accuracy, i.e. closeness to the examined amount (Wodarski 2009).

There are formulas determining the necessary number of experiments, but in this paper, analysis of convergence graphs of the examined estimators was used.

The strong law of large numbers states that the examined estimators converge with the unknown expected values when $n \to \infty$.

For the EK(X) estimator, the average of the sample ($\hat{\theta}$) was used. The experiment was repeated for all twelve months. The following results were obtained: expected values of term receivables percentage (to be exact, the values of their estimators) in the subsequent months were as follows (table 1) and average share of term receivables was 93.61%.The estimator of the expected value of the cost of all rebates allowed in the whole year was 5,388,813.41 PLN, which is ca. 8% of the mine's receivables and 41% of overdue receivables.

The graphs of the estimators ($\hat{\theta}n$) of the value ENt(X) for each month (and $n \in [1,100]$) as well as the graph of the expected value of rebate cost in the whole year EK(X) (Trzaskuś-Żak, 2013).

The graphs of estimators ($\hat{\theta}n$) for the expected values of term receivables ENt(X) in particular months are presented on fig. 1–12, and the graph of the estimator ($\hat{\theta}n$) for the expected value of rebate costs in the whole year EK(X) is shown on figure 13.

Graph of the estimator ($\hat{\theta}n$) for the expected value EK(X) of rebate costs in the whole year is shown on figure 13.

Estimators' values, both for particular months and for the total cost of allowed rebates, quickly get close to the limit values (Fig. 1–13). Thus, there is no need to conduct a larger number of random experiments.

The Conclusion

To summarise, the expected value of term receivables assumes values over 80% in every month and the annual average is 93.61%. The expected value of total cost of the allowed rebates is 5,388,813.41 PLN. Thus, it can be expected that with the adopted assumptions the total cost of allowed rebates determined using the Monte Carlo method should not exceed the anticipated amount of 5.5 million PLN. Average monthly number of rebates was also determined. In the analysed example, it was 42.

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Literatura – References

1. Czekaj J., Dresler Z. 1998. Zarządzanie finansami przedsiębiorstw. Podstawy teorii. Warszawa: Wydawnictwo Naukowe PWN, s. 274
2. Fuksa D. 2006. Wpływ uwarunkowań na politykę cenową przedsiębiorstwa. Szkoła Ekonomiki i Zarządzania w Górnictwie. Krynica, s. 127–132
3. Fuksa D. 2010, Polityka opustów cenowych na przykładzie przedsiębiorstwa górniczego, Komputerowo zintegrowane zarządzanie, Oficyna Wydawnicza Polskiego Towarzystwa Zarzadzania Produkcją. Opole, Vol.1, s. 440–445
4. Magda R., Wożny T., Kowalczyk B., Głodzik S., Gryglik D. 2002. Racjonalizacja modelu i wielkości kopalni węgla kamiennego w warunkach gospodarczych początku XXI wieku. Kraków: AGH University of Science and Technology Press
5. Kłak B. 2006. Skutki ekonomiczno-podatkowe wykorzystania rabatu w procedurach windykacji należności przedsiębiorstwa górniczego. Doctoral dissertation. Kraków: AGH University of Science and Technology in Kraków, Faculty of Mining and Geoengineering
6. Kopczewska K., Kopczewski T., Wójcik P. 2009. Metody ilościowe w R. Aplikacje ekonomiczne i finansowe. Warszawa: Wyd. CeDeWu, s. 646
7. Kreczmańska-Gigol K. [red.] 2010 – Kreczmańska-Gigol K. Aktywne zarządzanie płynnością finansową przedsiębiorstwa. Warszawa: Difin, s. 399
8. Law on VAT, Journal of Laws of 2004 No. 54 item 535, as amended
9. Michalski G., 2014. Strategiczne zarządzanie płynnością finansową w przedsiębiorstwie. Warszawa: Wyd. CeDeWu , s. 221
10. Nowak E. 2009. Zaawansowana rachunkowość zarządcza. Warszawa: PWE, s. 337
11. Panfil M. 2004. Zarządzanie należnościami w małym i średnim przedsiębiorstwie. Radom: Polska Agencja Rozwoju Przedsiębiorczości, s. 144
12. Portalska A., Kornatowicz O. 2003. Odzyskiwanie należności pieniężnych. Warszawa: C. H. Beck, s. 446
13. Sierpińska M., Wędzki D.1998. Zarządzanie płynnością finansową w przedsiębiorstwie. Warszawa: Wydawnictwo Naukowe PWN, s. 275
14. Trzaskuś-Żak B., Żak A. 2013. Binary linear programming in the management of mine receivables. Archives of Mining Sciences, Vol.58 Issue: 3, p. 941–952
15. Trzaskuś-Żak B., Czopek K. 2013. Optimisation of receivables management in a mine, using linear programming. Archives of Mining Sciences, Vol.58, Issue: 2, p. 541–550
16. Trzaskuś-Żak B., 2013, Receivables management supported by mathematical methods on an example of chosen mine. Dissertations Monographs No. 280, Kraków: AGH University of Science and Technology Press, s. 208
17. Wodarski K. 2009, Zarządzanie ryzykiem w procesie planowania strategicznego w górnictwie węgla kamiennego. Monography No. 245, Gliwice: Silesian University of Technology Press

Optymalizacja kosztu rabatów udzielonych przez kopalnię odkrywkową surowców skalnych za pomocą metody Monte Carlo

W artykule zaproponowano wykorzystanie metody Monte Carlo do symulacji kosztów rabatów udzielonych przez analizowaną kopalnię odkrywkową surowców skalnych. Udzielanie rabatów przez przedsiębiorstwa swoim odbiorcom produktów lub usług jest jedną z metod zarządzania należnościami.
W niniejszym artykule Metoda Monte Carlo została zastosowana w celu wyznaczenia oczekiwanej wartości kosztu proponowanych przez kopalnię rabatów, poprzez zastosowanie odpowiedniego algorytmu działań. Aplikację opracowanej metody przedstawiono na danych kopalni odkrywkowej i jej wybranych odbiorców. Selekcja odbiorców oparta została na dużej liczbie transakcji miesięcznych.

Słowa kluczowe: zarządzanie należnościami, rabaty, metoda Monte Carlo