The Trinomial Model in the Valuation of Mining Investment Projects

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Abstract

The present paper is dedicated to the valuation of mining investment projects by means of real options (ROV) using the trinomial tree. The paper consists of four parts. The introduction sets out the objective of the work. The next section presents the general assumptions of the valuation of real options by means of the trinomial tree. The third part of the article shows a brief example of calculation using the methodology presented in chapter 2. The final chapter provides a summary of the present research. The article ends with a list of literature used in its preparation.

Keywords: real options, trinomial tree, investment valuation, mining

Introduction

Making a decision about starting an investment is an extraordinarily difficult task. In the case of mining investment projects this is even more difficult due to the associated high capital expenditures. The decision is usually made under the conditions of high uncertainty (connected to the generation of both internal and external factors affecting the investments). Before making a final decision about commencing an investment process, those in charge of deciding try to estimate the economic and financial value of such an endeavour. Various kinds of analyses are used for that purpose (determining both the value and the risk of a given investment). The discount methods are included among the basic methods determining the value of an investment. The use of those methods determines the value of the project for a point in time of t=0 (for the day of the analysis), its results indicating whether an investment should be attempted or given up. The real options valuation method (ROV) is more flexible for decision making. It enables the investor to assess whether in the case of e.g. unfavourable conditions during the time of t=0 it is preferable to refrain from making a decision about investing. In order to calculate the value of the real option one can use the continuous Black-Scholes model (BS) or discrete models such as the bi- and trinomial model. Considering the fact that e.g. in the waiting option the investment may be initiated at any time (the American option), it is more justified to use the discrete models. The continuous model presents the values of the European option, which is the one that can be implemented when the options expire. The purpose of the present paper is to present calculations involving the value of the waiting option using a trinomial model.

The trinomial model in real options valuation

A real option may be valuated using a continuous model (the Black-Scholes model (BS)), if we are dealing with the European option, or using a discrete model in the form of bi- or trinomial trees. In the case of valuating an investment with the possibility of postponement (of the decision to attempt the investment), the discrete model should be used. The trinomial model will be discussed in this part of the paper. The construction of the trinomial model is similar to a binomial model; the so-called arbitrage probability should be also estimated – meaning the probability of an increase or decrease in an underlying asset, along with the internal value of the option in the second stage and the total value of the option in the third and final stage. The primary difference between these models involves the fact that in the case of a trinomial model at the given moment t there are three manners in which an underlying asset can progress: it can increase, decrease or remain steady. The rates of increase and decrease determine the relations 1,2. If an underlying asset remains steady, the rate equals 1 (Sojda A. 2013).

$$u = e^{\lambda \sigma \sqrt{\Delta t}} \tag{1}$$

$$d = e^{-\lambda \sigma \sqrt{\Delta t}} \tag{2}$$

where:
u – the underlying asset's rate of increase
d – the underlying asset's rate of decrease
σ – variability of the underlying asset
Δt = T/t
T – the number of years until an option expires
t – the number of subperiods (steps)

In formulae 1 and 2 the parameter λ is selected in such a way that the probabilities cannot be negative.

The arbitrage probability is estimated by means of formulae 4,5,6 (Sojda A. 2013).

$$p_u = \frac{1}{2\lambda^2} + \frac{1}{2}\frac{\mu}{\lambda\sigma}\sqrt{\Delta t} \quad (4)$$

$$p_d = \frac{1}{2\lambda^2} - \frac{1}{2}\frac{\mu}{\lambda\sigma}\sqrt{\Delta t} \quad (5)$$

$$p_m = 1 - \frac{1}{\lambda^2} \quad (6)$$

where:
p_u – the probability of increase
p_d – the probability of decrease
p_m – the probability of remaining steady
μ – estimated according to formula 7

$$\mu = r - \delta - \frac{\sigma^2}{2} \quad (7)$$

where:
δ – the cost of lost profits
r – the risk-free rate

During the second stage the internal value of the option is estimated in individual nodes according to formula (8) (Rogowski W. 2008).

$$c(V)_{w,i,n-t} = \max(V_{i,n-t} - X; 0) \quad (8)$$

where:
i = 1,….,Mn-t
M_{n-t} – the number of nodes of a binomial tree at the moment of n-t
X – the price of implementing an option
V – the value of an underlying asset in the node

Upon calculation of the internal value of an option one can enter the final stage – calculating the total value of the option. This estimate is assessed in accordance with formula (9) in the case of terminal nodes, and in accordance with formula (10) in the remaining cases [1,2].

$$c(V)_{i,n} = c_{w,i,n}(V) \quad (9)$$

$$c(V)_{i,n,t} = \max\left\{\begin{bmatrix} c(V)_{i.n-t+1,wzost} \\ p_u + c(V)_{i,n-t+1,spadek,} \\ p_d + c(V)_{i,n-t+1,bez\_zmian}\, p_m \end{bmatrix} e^{-r}; c(V)_{w,i,n-t}\right\} \quad (10)$$

After acquiring the results of each stage one can assess the moment when an option can be executed. If the total value of an option exceeds the internal value, the decision maker should refrain from implementing the option. The decision about implementation of an investment project can be made when both values are equal.

The use of the trinomial model for the valuation of a real option

The present part of the paper contains sample calculations based on the methodology presented in the previous chapter. In order to conduct the analysis, it has been assumed that a mining company holds a concession for mineral extraction issued for a period of 5 years. During this time the mine must start investing in a new mineral deposit. At this moment the net present value of cash flows (NPV) is negative. The profits which can be generated by a company from the investment add up to 80 mln zlotys (the value of the underlying asset); considering the necessary investment expenses amounting to 100 mln zlotys (the price of executing the option) the NPV equals -20 mln zlotys. Negative cash flows may be caused by either too high costs or by a disadvantageous market price – the conducted analysis focused on the NPV as the value of the project at the moment of t=0. People in charge of a mining company do not want to give up the possibility to extract a mineral deposit and consider postponing the investment, waiting for the conditions (internal or external) to change. To this end, a valuation of the investment will be performed with a waiting option during the validity of the concession using a trinomial model. For the day when the analysis is being conducted the variability of an underlying asset amounts to 25%. The adopted risk-free rate equals 3% and the cost of the lost profits is 8%. The timespan of the analysis covers the duration of the concession which lasts 5 years. The initial parameters of the analysis estimated based on the adopted assumptions are presented in table 1.

According to the procedure presented in the previous chapter, the value of the investment project was the first thing to be calculated. The calculations were conducted based on the adopted assumptions presented in table 1. The construction of a trinomial tree for this stage is presented in figure 1.

The second step when assessing the value of the option is the calculation of its internal value. The graphic form of the result depicting this value is presented in figure 2.

At the time point t=0 the internal value of the option equals zero – this results from the fact that

Tab. 1. The initial parameters for performing an analysis of the real options valuation involving postponement of the investment (source: own research)

Tab. 1. Parametry wstępne do przeprowadzenia analizy wyceny opcji realnej na opóźnienie inwestycji (źródło: badania własne)

Parameter	Value
The value of an underlying asset [mln zlotys]	80
The price of implementing an option [mln zlotys]	100
The number of years	5
The number of steps	5
The risk-free rate	3.00%
The cost of lost profits	8.00%
Standard deviation (sigma)	25.00%
u	1.28
d	0.79
p	0.34
1-p	0.66
lambda λ	1.4

0	1	2	3	4	5
					469
				329	329
			231	231	231
		162	162	162	162
	114	114	114	114	114
80	80	80	80	80	80
	56	56	56	56	56
		39	39	39	39
			28	28	28
				19	19
					14

Fig. 1. A trinomial tree determining the value of the investment project

Rys. 1. Drzewo trójmianowe określające wartość przedsięwzięcia inwestycyjnego

0	1	2	3	4	5
					369
				229	229
			131	131	131
		62	62	62	62
	14	14	14	14	14
0	0	0	0	0	0
	0	0	0	0	0
		0	0	0	0
			0	0	0
				0	0
					0

Fig. 2. The internal value of the option for a trinomial model

Rys. 2. Wartość wewnętrzna opcji dla modelu trójmianowego

0	1	2	3	4	5
					369
				229	229
			131	131	131
		62	62	62	62
	21	20	18	16	14
6	5	4	3	2	0
	1	1	0	0	0
		0	0	0	0
			0	0	0
				0	0
					0

Fig. 3. The total value of an option

Rys. 3. Całkowita wartość opcji

profits from the implementation of the project are lower than the amount of investment expenses. In order to determine whether it is profitable for a company to wait with the implementation of the investment, one should calculate the total value of the waiting option. According to the assumptions presented in the previous chapter, this value is presented by a trinomial tree shown in figure 3.

The value of flexibility for the conducted sample calculation amounts to 6 mln zlotys. The option involving the postponement of an investment has its value and it is therefore justified not to refrain from the investment. As indicated by figures 3 and 2, the total value of the option at the time point of t=0 is higher than the internal value of the option, therefore during the time of t=0 the investment should not be attempted. According to the presented rule, the investment should not be attempted during the time of t=1 either. The results of the analysis indicate that such an investment (according to the adopted assumptions) can be attempted in year 2, since during this period the total value of the option equals its internal value. The possibility of an investment in year 2 refers only to a situation when the profits from the investment (without the value of the investment expenses) are at least 162 mln zlotys.

Summary

The use of a trinomial model for the valuation of mining investment projects is justified. Unlike the traditional method (NPV) it does not rule out a given investment, but it examines whether it is profitable for an investor to postpone the investment.

Literatura – References

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Model trójmianowy w wycenie górniczych projektów inwestycyjnych

Niniejszy artykuł został poświęcony wycenie górniczych projektów inwestycyjnych przy użyciu opcji realnych (ROV) z wykorzystaniem drzewa trójmianowego. Artykuł składa się z czterech części. We wstępie określono cel pracy. W kolejnej części przedstawiono ogólne założenia wyceny opcji realnej przy użyciu drzewa trójmianowego. W trzeciej części artykułu przedstawiono krótki przykład obliczeniowy z wykorzystaniem metodologii zaprezentowanej w rozdziale 2. Ostatni rozdział stanowi podsumowanie niniejszego opracowania. Artykuł został zamknięty spisem literatury użytej do napisania opracowania.

Słowa kluczowe: opcje realne, drzewo trójmianowe, wycena inwestycji, górnictwo