

# **One-Dimensional and Two-Dimensional Analyses** of Hard Coal Separation in a Jig

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### Summary

Grained material characterises diversity because of physical properties, physicochemical properties and geometrical properties decide upon feed applicability of a beneficiation type. One-dimensional analyses are not enough for complete description of raw materials. Two-dimensional and multidimensional analyses are more and more popular. The most important properties of hard coal are: size, density and ash content. This paper presents an analysis and an appraisal of grained hard coal separation efficiency in a jig. Partition curves of a feed and size fractions are made. The probable error Ep is used to assess and compare the efficiency of gravity beneficiation of separated fractions. Further, two-dimensional empirical cumulative distributions and an empirical partition surface are analyzed in the following paper.

Keywords: coal, multidimensional analysis, partition curve, partition surface

# Introduction

Statistical methods together with the mathematical description are a common element of material and processes analysis in mineral engineering. Mineral characteristics and the procedure of mineral processes have a nature of random variables and functions, which are more and more often applied in various scientific fields. Wherever it can be observed an analogy with separation and graining of the examined material, it can be introduced partition numbers, curves and partition surfaces [Brożek and Surowiak, 2006; Brożek and Surowiak, 2010; Li et al., 2013; Niedoba, 2013b].

It must be said that the conditions of mineral beneficiation in processing plants are far from being perfect and stable. The process disturbances are first and foremost connected with the large scale of the partition. However, it should not be ignored other particles parameters, including particle density, shape, porosity, and others, for instance mutual interactions or turbulences [Mishra and Mehrotra, 1998; Surowiak, 2013; Niedoba, 2013a].

The characteristics of the particles, such as size, density, or the valuable ingredient or mineral substance content, have a substantial influence upon the partition procedure. It is of great importance to use two-and multi-dimensional distribution functions in order to describe accurately coal beneficiation process as most of the processing operations do not depend only upon one parameter [Olejnik et al., 2010; Rao et al., 2003; Sztaba, 2003].

## The aim and the spectrum of experiment

The following paper is based upon the investigation conducted on hard coal samples taken from the mechanical coal preparation plant in Poland. The sample has been taken in order to examine the partition determinants of the grained material beneficiated in a water pulsatory jig.

The analysis of partition quality was conducted on the basis of partition curves and partition indices, being the accuracy determinants. Accordingly, the main aims of the paper are the partition efficiency assessment in every size fraction, and the multi-dimensional analysis of coal partition, which was conducted by defining two-dimensional empirical cumulative distributions for the feed and the concentrate. Moreover, this paper focuses on defining the empirical partition surfaces.

# Experiment

The samples of coal enabled to conduct an analysis of the factors influencing the coal separation in a three-product industrial jig manufactured by Allmineral. The representative samples of jig concentrate and tailings have been collected in one of the Polish hard coal mines. The samples were characterized by granulation below 20 mm.

For the laboratory research representative samples have been dispensed. Then, an analysis of particle size distribution of the concentrate and tailings, and density analysis were conducted. The density analysis was performed in zinc chloride suspensions of various densities. The cases of the following size fractions were investigated in the following work: 1-3.15, 3.15-6.3, 6.3-10, 10-12.5, 12.5-16, 16-18, 18-20 mm. The research was conducted in accordance with the appropriate and currently binding legal norms (such as PN-G-04559:1997, PN-R-04032: 1998 and PN-79/G-04533). In order to minimize mistakes, the ash analysis has been repeated.

#### **Results and discussion**

The assessment of the coal beneficiation in an industrial jig was made on the basis of the partition assessment index, which is the probable error Ep defined for the whole feed and for every narrow size fraction [Niedoba, 2013b; Teffo and Naude, 2013; Tumidajski and Saramak, 2009]. The coordinates of partition curves in case when particle density is separation feature of separation number  $\tau(\rho)$  were calculated from the following formula:

$$\tau(\rho) = \gamma_k \frac{f_k(\rho)}{f_n(\rho)} \tag{1}$$

where:

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 $\gamma_k$  – the percentage number of concentrate except for size fraction 0-1 and >20 mm;  $f_k(\rho)$  – distribution density function of the particles density in concentrate;  $f_n(\rho)$  – distribution density function of the particles density in feed.

The information was hereby obtained about the output of particles with the specific density in concentrate. Below a chart (Fig. 1) showing partition curve  $\tau(\rho)$  for the concentrate is presented. Ignoring the size fraction below 1 mm caused that the percentage number of tailing at the point of 51.8% was obtained; hence, the concentrate constituted 48.2%.

In order to draw the partition curves  $\tau i(\rho)$  in each size fraction it was needed to determine the percentages of tailing and concentrate content in the feed. The values of the feed were calculated from data of concentrate and tailings. Data on feed obtained from an equation balance. The partition curves for every concentrate size fractions are presented on Fig. 2.

Considering the efficiency of partition in every fraction, it can be concluded that during the beneficiation disturbances occurred, the existence

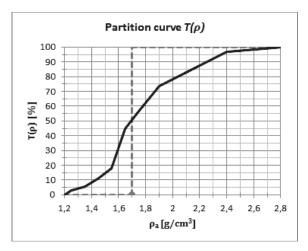


Fig. 1. Partition curve  $\tau(\rho)$  for all tailings (straight line) and the perfect partition curve (dotted line) Rys. 1. Krzywa rozdziału  $\tau(\rho)$  dla całego odpadu (linia ciągła) i idealna krzywa rozdziału (linia przerywana)

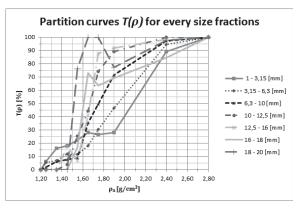


Fig. 2. Partition curves  $\tau_i(\rho)$  for every size fraction Rys. 2. Krzywe rozdziału  $\tau_i(\rho)$  dla poszczególnych klas ziarnowych

of which is confirmed by the partition curves  $\tau i(\rho)$ . The turbulences during the process are connected with its flow nature in industrial conditions. Moreover, the separation of the particles is a random phenomenon, which is caused by the already mentioned, impact demolition.

For the analyzed fractions, the density values corresponding to the partition number equal to 50% agree only in size fractions 10-12.5 and 12.5-16 mm and their cut point is equal 1670 kg/m<sup>3</sup>. In the remaining cases they amount to numbers between 1520 and 2080 kg/m<sup>3</sup>. It means that the probability of displacement to the concentrate of the particles with the same density is diverse in various size fractions.

There are different parameters to indicate a departure of the partition curves from the ideal shape. For the partition curves the most frequently use is Ep (probable error) defined as half of difference between the sizes at which 75% and 25% recoveries are observed:

$$E_p = \frac{|\rho(\tau = 75\%) - \rho(\tau = 25\%)|}{2}$$
(2)

where:

 $\rho(\tau=75\%)$  – density corresponding to the partition number of 75%;  $\rho(\tau=25\%)$  – density corresponding to the partition number of 25%.

Besides  $E_p$ , there are many others parameters (imperfection, sharpness etc.) aiming to represent the partition curve and to evaluate the efficiency of the separation process. The values of  $E_p$  for size fractions are presented on Fig. 3.

Taking into account  $E_p$  value, it can be claimed that the most precise partition is in size fractions: 18-20, 10-12.5 and 12.5-16 mm. The efficiency of partition in the remaining size fractions is worse

and the most imprecise value riches in 1-3.15 mm size fraction.

Comparing the partition curve shape and the value of probable error in size fraction 18-20 mm, it can be noticed that findings are incommensurate. The shape of curve indicates an advent of interferences and disturbances during the separation. Nonetheless, the low value of probable error disclaims that conclusion. Fig. 2 and 3 are showing this interrelation.

**Multidimensional analysis of hard coal separation** Multidimensional analyses of grained materials are based on a thesis that separated material is distinguished by multidimensional vector of particles properties W.

$$W = [W_1, W_2, \dots, W_n]$$
(3)

where:

 $W_1 = D$  – particle size;  $W_2 = P$  – particle density;  $W_n$  – other properties, for example ash content.

Separated, grained material is a set of random variables, and it is possible to describe it in different empirical distribution functions of random variables and create a multidimensional distribution of a set of properties.

However, as it has been already mentioned, the process of separation in different types of hard coal is mass and having flow nature. So, the gravity separation is reliant on two properties of particles. Particle size and density have an influence on the effect of separation [Brożek and Surowiak, 2010; Niedoba, 2013b; Olejnik, 2010; Rao, 2003; Sztaba, 2003]. In carrying out two-dimensional analysis grained material, taking into account the above mentioned characteristics, graphs no. 4, 5 and 6 were created.

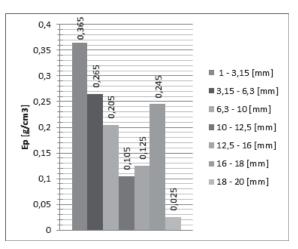
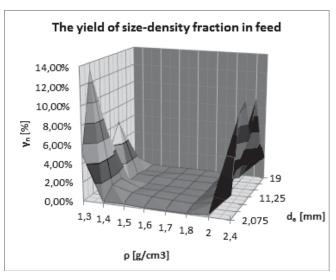
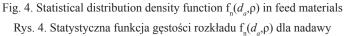
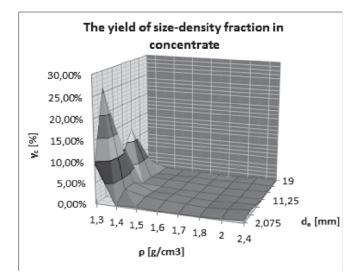
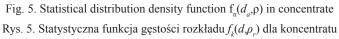


Fig. 3. Graph of  $E_p$  and size fractions Rys. 3. Wykres wartości  $E_p$  w zależności od klas ziarnowych









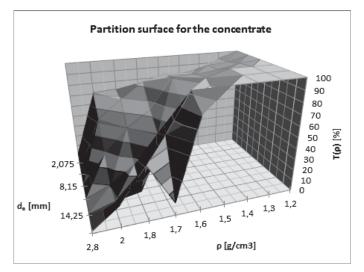


Fig. 6. The partition surface  $T=T(\rho, d_a)$ Rys. 6. Powierzchnia rozdziału  $T=T(\rho, d_a)$ 

Fig. 4. shows the yield of grained materials in every fraction. In view of the disregard of extreme size fractions and probable mistakes during sample collection and laboratory research, it can be assumed that the yield of concentrate and tailings differs from the actual results a little.

The shape of surface which is empirical distribution density function  $f_n(d,\rho)$ , indicates that the particles with density below 1300 kg/m<sup>3</sup> and with the tailings above 2000 kg/m<sup>3</sup> constitute the biggest share in feed. In the density fraction of below 1300 kg/m<sup>3</sup> the particles with the size of 1-16 mm dominate. Whereas, the size fraction 3.15-16 mm constitutes the biggest share in the density fraction of 1300-2000 kg/m<sup>3</sup>. The density fraction of 1300-2000 kg/m<sup>3</sup> makes the least visible share, constituting 14.2% of the whole feed.

The following chart (Fig. 5) presents the relationship between the main characteristics of the separation of grained material and the yield of fraction in concentrate.

Looking at the chart, it can be assumed that the biggest share constitutes the material with density below 1400 kg/m<sup>3</sup>. It makes about 85% of all the concentrate, whereas the remaining fractions have less significance, and its share constitutes 15%.

In this paper the partition surface of the grained material, taking into account the density and the size of particles, has been described. Having a second look at the above chart, it can be observed that the extreme size fractions, those below 1 mm and bigger than 18 mm, have been omitted. Taking into consideration the grained material of 1-18 mm, it has been stated that the yield of concentrate constitutes 48.21%. Fig. 6 illustrates the relationship of probability of particles transport to the concentrate, taking into consideration two partition characteristics  $(d,\rho)$ .

Analyzing the whole grained material separated only in terms of density, the probable error  $E_p$  is 253.20 for the feed. This value can be observed having chart no. 1 as the basis. Whereas, having a fixed particle size, crossing the partition surface from the chart no. 6 with the  $d=d_0$  plane, it can be obtained partition curves for every size fraction. For each of those curves it can be assessed the value of the probable error  $E_p$ . In this way, it can be obtained a curve illustrating the relation of  $E_p$  and the particle size.

#### Conclusions

I. During the sample collection from a device at work it is hard to preserve the absolute representativeness of samples (the lack of data in some density fractions). II. The gained ash content in every density fraction (irrespectively from the partition products) in various size fractions are similar (about 50%), which becomes evidence for the accurateness of the analyses conducted. One exception appears in the ash content in concentrate in the size fraction of 18-20 mm (about 32.60%). The most probable cause of such an error is giving too much material simultaneously in one analysis.

III. Analyzing the efficiency of separation on the basis of the partition curve shapes for every size fraction, it can be concluded that during this process there have occurred some interferences connected with the mass nature of industrial conditions. For the scrutinized size fractions, the set point of 1670 kg/m<sup>3</sup> corresponding to the partition number of 50%, are the same merely in two size fractions: 10-12.5 mm and 12.5-16 mm. In the remaining size fractions the values of cut points are between 1520 and 2080 kg/m<sup>3</sup>.

IV. From the analysis of probable error for every size fraction it can be concluded that the partition efficiency is the biggest in fractions 10-12.5 mm and 12.5-16 mm, and it corresponds to the previous conclusions. Whereas, in size fraction 18-20 mm, it can be assumed (on the basis of the value of  $E_p$ ) that the partition efficiency is the most significant, which stands in opposition to the conclusions from the partition curve shape for this fraction.

V. From the two-dimensional analysis of size-density fraction yield for the feed, it can be concluded that the particles with density below 1300 kg/m<sup>3</sup> and with the tailing above 2000 kg/m<sup>3</sup> constitute the biggest share in feed. In the density fraction of below 1300 kg/m<sup>3</sup> the particles with the size of 0-16 mm dominate. Whereas, the size fraction 3.15-16 mm constitutes the biggest share in the density fraction of above 2000 kg/m<sup>3</sup>. The density fraction of 1300-2000 kg/m<sup>3</sup> makes the least visible share (constituting altogether 14.2% of the whole feed).

VI. On the ground of the chart, it can be assumed that the biggest share constitutes the material with density below 1400 kg/m<sup>3</sup>. It makes about 85% of all the concentrate, whereas the remaining fractions have less significance, and its share constitutes 15%. VII. The partition surface shape confirms occurrence of the turbulences during the separation. Size fractions of the same density have got various values of partition numbers. It follows that the efficiency of separation is dependent on size and density of particles. The same conclusions are included in points III and IV.

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Jedno- i dwuwymiarowa analiza rozdziału węgla kamiennego w osadzarce pulsacyjnej Materiał uziarniony charakteryzuje różnorodność ze względu na własności fizyczne, fizykochemiczne czy geometryczne, które decydują o podatności nadawy na określony sposób wzbogacania. Analizy jednowymiarowe często nie wystarczą do pełnego opisu materiału uziarnionego, dlatego powszechne stają się analizy dwu- i wielowymiarowe. W przypadku węgla kamiennego istotne są trzy właściwości: wielkość ziarna, gęstość ziarna i zawartość popiołu. W niniejszej pracy przedstawiono analizę i ocenę dokładności rozdziału ziaren węgla kamiennego w osadzarce miałowej pulsacyjnej. Wykreślono krzywe rozdziału nadawy oraz poszczególnych klas ziarnowych, a także obliczono rozproszenie prawdopodobne w celu porównania dokładności rozdziału ziaren z wyodrębnionych klas. Dalszej analizy dokonano w oparciu o dwuwymiarowe dystrybuanty empiryczne nadawy i koncentratu, a także o empiryczną powierzchnię rozdziału.

Słowa kluczowe: węgiel, analiza wielowymiarowa, krzywa rozdziału, powierzchnia rozdziału