Identification of Dynamic Model of Coal Flotation Process on the Basis of Smoothed Step Response

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Summary

The paper presents the results of modelling of dynamic properties of coal flotation as a control object determined by the use of the smoothed step response that is ash content in tailings to the step change of the chosen input signal (amount of flotation reagent). The measurement data of ash content in tailings and reagent flow rate from the industrial test were used to calculate the parameters of the model. The theoretical basis of the calculation of the model parameter values based on knowing step response of the first order inertia with the time delay were presented. The values of parameters of dynamic model of coal flotation were determined by the method based on the equations of step response of the object using the smoothing filter in the calculation example. The results were compared with calculation results achieved by the least squares method.

Keywords: coal flotation process, identification, dynamic model, digital filter, smoothing

Introduction

The coal flotation process as a control object is a nonlinear dynamic object of multiple inputs and multiple outputs. The most important input parameters of flotation process are: feed flow rate Q_n with the ash content A_n and solids concentration in the feed K_{cs}, flotation reagent flow rate V_o, air flow rate to the aeration Q_a and the level of the slurry in the flotation cell h. Output parameters include: the yield concentrate W_k, ash content in the concentrate A_k, the yield of tailings W_o, ash content in the waste tailings A_o. The control signals in the flotation process are reagent flow rate, the level of slurry in the cell, air flow rate to the aeration. Due to the random variation feed parameters are disturbances in the flotation process. In the domestic industrial installation the part of the input and output signals are measured during the process. The only qualitative parameter measured during the process by the use of MPOF sensor is the ash content in the tailings [1]. Solids concentration and flow rate feed usually are measured from the feed parameters.

Step response method belongs to the basis method for the identification of the dynamic models. That method includes the experiment in which step change of the input signal of the system is set (in this case – step change of reagent flow rate) and the changed output signal is measured (step response of the ash content in the tailings). The experiment is one of the stages of the identification of the dynamic model whose purpose is to record the measurement data. The next stage is to select the structure of the dynamic model. On the basis of many industrial tests [2][3][4] it was shown that model of the first order inertia with time delay sufficiently accurately describes the dynamic properties of coal flotation process with the control input V_o and output A_o. When the model structure is known its parameters are calculated based on recorded time series data. It involves choosing an appropriate estimation method. It is one of the most important stages of identification as the quality of the developed model (in the sense of assumed criterion) to a large degree depends on the choice of the estimation method. Description of the dynamic properties of the flotation process in the form of mathematical model is necessary not only to create simulation models but also design system and develop automatic control algorithm of this process. There is therefore a need for a search and improvement on identification methods in order to determine models that allow for the more precise description of dynamic properties of the industrial coal flotation process.

The article presents a method for computing dynamic model parameters based on the smoothed step response used for the modelling of coal flotation process. The digital polynomial filter that smoothes by the approximating polynomial of the 2^{nd} degree was used to smooth dynamic characteristic (step response). This paper presents the results of modelling of flotation process using the proposed method, industrial measurement data was used as empirical data for research. Comparative analysis of the results of the proposed method with the results obtained by the least squares method was also carried out.

Determining model parameters based on step response

Dynamic properties of the flotation process as an object of one control input (flotation reagent flow rate) and one output (ash content in the tailings) can be presented in the form of a model of the first order inertia with time delay [2][3][4]. Step response of the structure of the first order inertial element with time delay to change step of input signal is presented by the formula:

$$y(t) = y_u \cdot \left(1 - e^{-\frac{(t-\tau)}{T}}\right) \tag{1}$$

where:

y_u – value of output signal in steady state ($dy/dt = 0$ and $t > \tau$),

T – time constant of the system,

τ – time delay.

Example of step response described by the equation (1) for zero initial conditions is presented in Fig. 1

When the output signal in steady state (y_u) and values of two points (in unsteady state), for example values of step response of the object for the time t_v, t_w are known, then system of two equations with two unknowns τ and T can be written as:

$$\begin{cases} y(t_v) = y_u \cdot \left(1 - e^{-\frac{(t_v-\tau)}{T}}\right) \\ y(t_w) = y_u \cdot \left(1 - e^{-\frac{(t_w-\tau)}{T}}\right) \end{cases} \quad (2)$$

The solution of system of equations (2) with regard for the unknown time delay (τ) and time constant (T) are presented by the formulas:

$$\tau = \left(t_w \cdot \ln\left(1 - \frac{y(t_v)}{y_u}\right) - t_v \cdot \ln\left(1 - \frac{y(t_w)}{y_u}\right)\right) \cdot \left(\ln\left(1 - \frac{y(t_v)}{y_u}\right) - \ln\left(1 - \frac{y(t_w)}{y_u}\right)\right)^{-1} \quad (3)$$

$$T = (\tau - t_v) \cdot \left(\ln\left(1 - \frac{y(t_v)}{y_u}\right)\right)^{-1} \quad (4)$$

Calculating values of unknown τ, T is possible when the step response (y_v, y_w) for the time (t_v, t_w) and value in steady state (y_u) have the same sign and the following conditions are fulfilled: $0 < t_v < t_w$ and $|y(t_v)| < |y(t_w)| < |y_u|$. As equations (3) and (4) show knowing value of the output signal in steady state and two points positioned on the exponential curve, presented in Fig.1 is sufficient to calculate unknown model parameters such as time constant, time delay. In order to check the precision of calculations, results should be compared to the results obtained for other pairs of points from the exponential curve (1). Presented equations (3) (4) refer to the case when the influence of disturbances on the object is minimal. If there is clear noise in the output signal then using the above equations requires the application of the proper filter, for example smoothing polynomial filter.

Smoothing of signal by the approximating polynomials

Smoothing is reducing higher frequencies from signal using the measurement data (from the past and future) in relation to the calculating point. In the used method the value of the next output sampling filter is calculated for the point being in the middle of the assumed time range approximating time series data by the polynomial of the 2nd degree. The calculations are performed on the basis of the earlier recorded (with sampling period T_s) measurement data. The filer was described in detail in literature [5][6][7][9][10]. Graphical interpretation of smoothing the point in the range ($2M+1$) of different widths was shown in Fig. 2. As it is shown in Fig. 2 the value of smoothing point at the given moment k = t/Ts = 0 is determined based on the number of samples symmetrically positioned on the timeline (2M+1) [5]. The value of the polynomial in the middle of the range (at the moment $k = t/T_s = 0$) is the point of smoothed time series. For the following successive moments (k+1, k+2, …) the analogous calculations are made, obtaining smoothed time series data.

Equation of the digital polynomial smoothing filter (the approximation by the polynomial of degree α in the range of width $2M+1$) can be presented by the formula:

$$y_f[i] = \sum_{k=-M}^{M} c_k \cdot y[i+k] \quad (5)$$

where:

$2M+1$ – width of the range positioned symmetrically in relation to the smoothing point,

T_s – sampling period,

c_k – coefficients of smoothing filter,

$y_f[i]$ – output signal of the filter in the moment of time iT_s.

The values of the particular coefficients of the polynomial c_k are calculated independently of the value of the measurement points of smoothing data series. For this purpose, the equation (6) is minimized. It allows to determine parameters d_w of polynomial of degree α for the range ($2M+1$) [10]:

$$J = \sum_{k=-M}^{M} \left(y[k] - \sum_{w=0}^{\alpha} d_w \cdot k^w \right)^2 \quad (6)$$

where:

d_w – parameters d_0, d_1, ..., d_α, of the approximating polynomial of degree α.

The values of the polynomial for the middle of the range ($2M+1$) are calculated based on d_0. However, the number and value of the coefficients c_k for the given α are not dependent on the value of measurement points but the length of the assumed range ($2M+1$). When the values of coefficients c_k are known then smoothing is performed by simple arithmetic operations on the measurement points that are in nearby position – determined by the range ($2M+1$) – of the

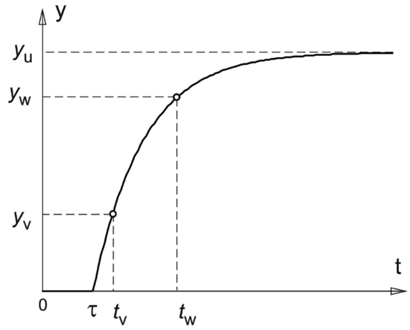

Fig. 1 Response of the first order inertia object with the time delay to the step change of input signal

Rys. 1 Odpowiedź obiektu o strukturze elementu inercyjnego pierwszego rzędu z czasem opóźnienia na skokową zmianę sygnału wejściowego calculating point. The polynomial smoothing filters allow to obtain the output time series of the filter without phase shift. For this reason their application in the identification algorithm of dynamic properties of the coal flotation based on the equations (3) (4) is justified.

Determination of parameters of the dynamic model of coal flotation on the basis of smoothed step response

In the step response method, the system passes from one point to another one in the steady state as the result of the changed step of the input signal. The condition for the proper conduct of the experiment in the case of objects of multiple inputs and multiple outputs is the constancy in time of the input signals (with the exception of the stimulating signal). Due to the nonlinearity of the static characteristics of the flotation process determining the parameters of the dynamic model is possible assuming zero initial conditions (the initial values of the input signal and the output are reduced to zero). It can be achieved by subtracting initial values: the amount of flotation reagent $V_o(0)$ and the ash content in the tailings $A_o(0)$ from the proper series data.

As regards the above comments for further consideration, following designations have been accepted:

- $u(t)$ – step change of the flotation reagent flow rate $u(t) = V_o(t) – V_o(0)$,
- $y(t)$ – change of the output signal (step response of the ash content in the tailings) $y(t) = A_o(t) – A_o(0)$,
- $y_f(t)$ – smoothed signal $y(t)$,
- $A_o(0)$, $V_o(0)$ – initial values of signals: the output and the input in the time $t(0)$, that is, before the start of the identification experiment.

Recording the signals from the particular measurement sensors is carried out every sampling period (T_s) in the industrial solutions of the control system and monitoring of the coal flotation. When the structure of the dynamic model of flotation as the object of the control input V_o and output A_o is known then the model parameters are determined based on measurement data by using one of the estimation method. As the result of using of the batch methods to estimate the parameters of the dynamic model, the calculated time delay is the multiplicity of the sampling period ($\tau = iT_s$). However the time delay (τ) is not the multiplicity of T_s when it is calculated on the basis of the equation (3). Due to the significant noise in the measurement signal from the optical ash meter, using the equations (3) (4) to calculate the parameters of the dynamic model of the coal flotation of one control input (flotation reagent flow rate) and one output (ash content in the tailings) based on the time series of the step response will lead to the significant and unacceptable errors making in this case this method useless. Using the method based on the equations (3) (4) is possible when the appropriate filter reducing noise in the measurement signal is applied in a way that allows the reconstruction of the useful signal. In case when the smoothing polynomial filter is used, model parameters (τ, T) are not calculated based on the strict measurement data but the signal obtained at the filter output. Then appropriated values of smoothing signal were introduced to equations (3) (4).

As a smoothing filter, a filter approximating by the 2[nd] degree polynomial for the width of the smoothing range ($2M+1$) was accepted, where M is a natural number from the assumed range [2, M_{max}]. A method for the identification of the dynamic model of the coal flotation using the smoothing filter is presented in Fig. 3. Assuming the degree of the approximating polynomial (the 2[nd] degree) the choice of smoothing filter parameter is come down to determine the appropriate value M from the assumed range. Because of the realization of the estimation of the model parameter on the basis of recorded measurement data, smoothing time series can be performed K – times at the given parameter M. Then the output signal of the filter is directed each time to its input in order to perform the next smoothing operation.

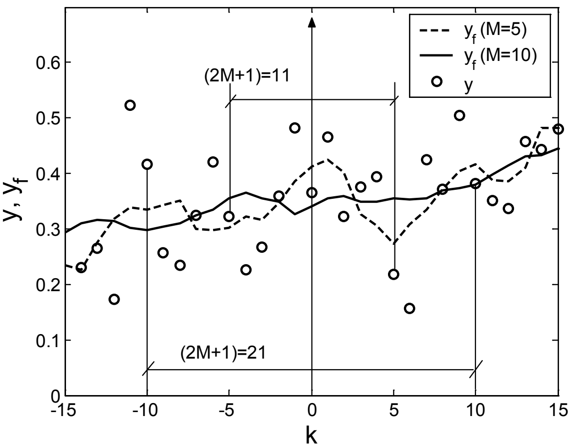

Fig. 2 Examples of the signals at the filter output realizing smoothing by polynomial of the 2[nd] degree of two widths against a background of the measurement data
y – measurement data, y_f - values of signal at the output smoothing filter

Rys. 2 Przykładowe przebiegi na wyjściu filtru realizującego wygładzanie wielomianem drugiego stopnia w przedziałach o dwóch szerokościach na tle danych pomiarowych
y – dane pomiarowe, y_f- sygnał na wyjściu filtru wygładzającego

Evaluating the accuracy of the model

The variance was taken for the evaluation of the fit of the model to the empirical data.

$$J_e = \sigma_e^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(e[i]\right)^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(y[i]-\hat{y}[i]\right)^2 \quad (7)$$

where:
$y[i]$ – output signal of the object (measured ash content in the tailings),
$\hat{y}[i]$ – output signal of the model,
$e[i]$ – error,
N – number of data being useful for the determination of the model parameters.

The smaller variance value the better fit of the model to empirical data is. In the case of the proposed method, the aim of the identification of the dynamic model of coal flotation by the use of smoothing the step response is to determine the model and filter parameters such that will minimize the criterion (7). That formulated condition can be written as follows:

$$\left(T,\tau,y_{fu},M,K\right) = \min\{J_e\} \quad (8)$$

Evaluation of the proposed method was carried out by comparing the achieved results with the estimation results of the model parameters by the least squares method. The least squares method was used to determine the parameters of the differential equation describing the dynamics of the object in the form:

$$y[i] = -a_1 \cdot y[i-1] + b_m \cdot u[i-m] \quad (9)$$

where:
$m=\tau/T_s$ – representing time delay.

The calculated parameters (a_1, b_m) of the difference equation (9) should be converted to parameters for the continuous-time model (τ, T).

Results of calculations

The measurement data recorded in the industrial installation of flotation process – that were presented in [3] – were used for calculations. The time series data of ash content in tailings were measured by ash meter MPOF (optic instrument) [1][8][11]. The feed parameters were monitored and their signals have constant values during the measurement (during the industrial test) - K_{cs} = (119±1)g/ dm^3, Q_a = (146±3) dm^3/s. The values of parameters M was chosen from the range [2, 12] – for the single smoothing and performing K-times (from 1 to K_{max} = 20) smoothing of signal from the filter for the particular M from the assumed range. The last computed point of smoothing signal at the filter output ($y_{fu}[N]$) was accepted for value of output signal in steady-state (y_u). The sampling period was equal T_s = 60s.

The values of the calculated parameters of the dynamic model of the flotation process are arranged in Tab. 1 and the time series are presented in Fig. 4. Presented results (Tab. 1) concern the choice:
1) parameter M of the filter that is the range width $2M + 1$, by the single smoothing signal ($K = 1$),
2) parameter M of the filter, by the smoothing time series K_{max} – times ($K = 20$),
3) times of the smoothing K, by assumed the narrowest range of smoothing ($M = 2$),
4) times of the smoothing K, by assumed the widest range of smoothing ($M = 12$)
5) both parameter M and times of smoothing K.

Values of equation coefficients ck of the smoothing digital filter (5) (approximation by polynomial of the 2nd degree) for particular cases from Tab. 1 were arrange in Tab. 2.

On the basis achieved results it was stated that the proposed determination method of the model parameters with the application of the smoothing of the step response using polynomial (α = 2) with exception of cases 3 and 4 from Tab. 1 provided better results (in the sense of the assumed criterion) than the last squares method. The best results was obtained by choosing M and K (Tab. 1 case 5). As the Fig. 4 shows the shape of output filter signal is equal to the shape of signal at the output model calculated on the basis of equations (3) (4). The proper choice of parameter M was more important than K–time smoothing (in assumed ranges

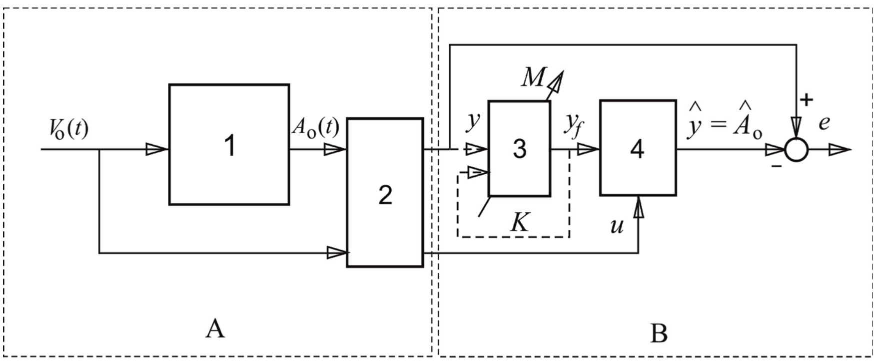

Fig. 3 Identification of dynamic model of coal flotation using smoothing of step response by smooth-ing polynomial filter
A) Experiment, B) Computing of T, τ, $y_f u$;
1 – coal flotation process, 2 – recording of time series data, 3 – smoothing filter, 4 – dynamic model

Rys. 3 Identyfikacja modelu dynamiki procesu flotacji węgla z wykorzystaniem wygładzania odpowiedzi skokowej obiektu za pomocą wielomianowego filtru wygładzającego
A) Eksperyment, B) Wyznaczenie T, τ, $y_f u$; 1 – proces flotacji węgla, 2 – rejestracja danych, 3 – filtr wygładzający, 4 – model dynamiki

Tab. 1 Identification results of dynamic model of coal flotation process

Tab. 1 Wyniki identyfikacji modeli dynamicznych procesu flotacji węgla

Case	Method	u [cm³/s]	Model parameters		J_e	Initial values		$y_f(t_v)$ $y_f(t_w)$ [%]	t_v t_w [s]	y_{fu} [%]	M	K
			T [s]	τ [s]		y_0 [%]	u_0 [cm³/s]					
1			202,8	195,8	0,7738			3,587 7,712	300 600	8,929	11	1
2			243,9	182,1	0,7162			3,585 7,667	300 600	9,353	5	20
3	1	1,00	143,5	218,9	0,9464	49,7	0,83	3,686 7,151	300 480	8,534	2	1
4			227,0	180,2	0,8509			3,661 7,524	300 600	8,929	12	1
5			213,4	197,2	0,6924			3,520 7,812	300 600	9,206	5	15
6	2		166,8	180,0	0,7919			-	-	8,915	-	-

1 – method based on equations (3) (4) and smoothing a signal, 2 – the least squares method

Tab. 2 Coefficients c_k (α = 2) for cases from Table 1

Tab. 2 Wartości współczynników c_k (α = 2) dla przypadków z Tablicy 1

case	M	Cases tab.1	c_0	c_1 $c_{(-1)}$	c_2 $c_{(-2)}$	c_3 $c_{(-3)}$	c_4 $c_{(-4)}$	c_5 $c_{(-5)}$	c_6 $c_{(-6)}$	c_7 $c_{(-7)}$	c_8 $c_{(-8)}$	c_9 $c_{(-9)}$	c_{10} $c_{(-10)}$	c_{11} $c_{(-11)}$	c_{12} $c_{(-12)}$
1	2	3	0,486	0,343	-0,086	-	-	-	-	-	-	-	-	-	-
4	5	2, 5	-0,084	0,021	0,102	0,161	0,196	0,207	-	-	-	-	-	-	-
7	11	1	-0,052	-0,026	-0,002	0,019	0,037	0,053	0,0671	0,078	0,087	0,093	0,097	0,098	-
8	12	4	-0,049	-0,027	-0,006	0,012	0,028	0,043	0,0554	0,066	0,075	0,081	0,086	0,089	0,090

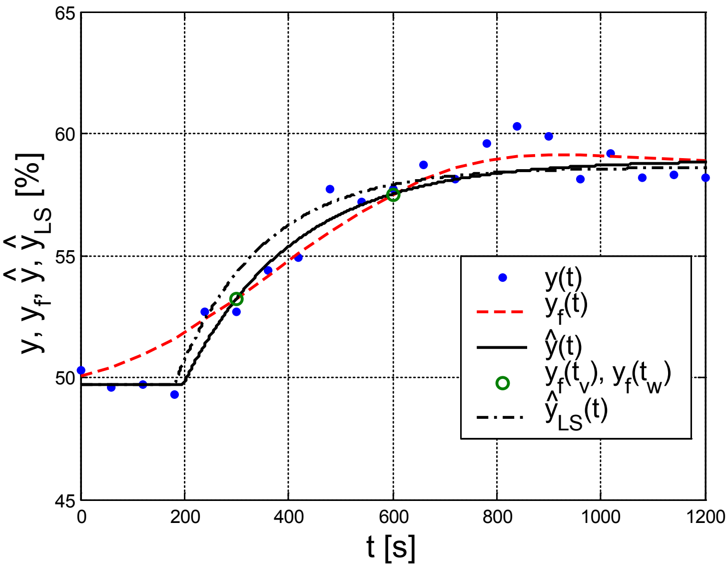

Fig. 4 Values of signal at the output smoothing filter $y_f(t)$ and response of ash content in tailings to step change reagent flow rate (from value 0,83 cm3/s to 1,83 cm3/s):
$y(t)$ – measurement data, $\hat{y}(t)$– signal at the output model determined by the method based on equations (3) (4) and smoothing step response (Tab. 1 case 5), $\hat{y}_{LS}(t)$ – signal at the output model determined by the least squares method (Tab. 1 case 6)

Rys. 4 Przebieg sygnału na wyjściu filtru wygładzającego $y_f(t)$ oraz odpowiedź zawartości popiołu w odpadach flotacyjnych na skokową zmianę natężenia przepływu odczynnika flotacyjnego (od wartości 0,83 cm³/s do 1,83 cm³/s)
$y(t)$ – dane pomiarowe, $\hat{y}(t)$ – sygnał na wyjściu modelu wyznaczonego metodą opartą na rów. (3) (4) i wygładzaniu odpowiedzi skokowej (Tab. 1 pkt. 5), $\hat{y}_{LS}(t)$- sygnał na wyjściu modelu wyznaczonego metodą najmniejszych kwadratów (Tab. 1 pkt. 6)

of both parameters) in the considering example. In cases in which *K* was being determined and the values *M* = 2 or *M* = 12 were assumed (Tab. 1 cases 3, 4), the obtained results were worse than in situation when parameter *M* was chosen by assumed multiplicity of smoothing *K* = 1 or *K* = 20 (Tab. cases 1, 2).

Conclusions

The choice of estimation method of parameters of dynamic model has an essential influence on its quality. The calculated results show that the method of determining of dynamic model of coal flotation using smoothed step response process by approximating polynomial of the 2nd degree allows to obtain better results (in the sense of the assumed criterion) than the least squares method. It can be because of the fact that time delay (τ) calculated using the formula (3) in general is not multiplicity of sampling period (T_s) what occurs when the least squares method is applied. It shows that the method based on equations (3) (4) and smoothed step response can be useful method for identification of dynamic model describing the influence of flotation reagent flow rate on the ash content in tailings. Difficulty of this method is the possibility of its application only relating to the step response systems of the structure of the first order inertia.

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Identyfikacja modelu dynamiki procesu flotacji węgla na postawie wygładzonej odpowiedzi skokowej

W artykule przedstawiono wyniki modelownia własności dynamicznych flotacji węgla kamiennego, jako obiektu sterowania, wyznaczone z użyciem wygładzonej odpowiedzi skokowej, tj. przebiegu zawartości popiołu w odpadach, na skokową zmianę wybranego sygnału wejściowego (natężenia przepływu odczynnika flotacyjnego). Do wyznaczenia modelu wykorzystano przemysłowe dane pomiarowe zawartości popiołu w odpadach flotacyjnych i natężenia przepływu odczynnika flotacyjnego. Podano podstawy teoretyczne obliczania wartości parametrów identyfikowanego modelu na podstawie znajomości charakterystyki skokowej obiektu inercyjnego pierwszego rzędu z opóźnieniem czasowym. W przykładzie obliczeniowym wartości parametrów modelu dynamiki procesu flotacji węgla wyznaczono metodą bazującą na równaniach odpowiedzi skokowej obiektu przy zastosowaniu filtru wygładzającego. Rezultaty porównano z wynikami obliczeń uzyskanymi metodą najmniejszych kwadratów.

Słowa kluczowe: proces flotacji węgla, identyfikacja, model dynamiki, filtr cyfrowy, wygładzanie