Using the Total Least Square Method by Deformation Strain Analyses of the Eastern Edge Eastern Tatras

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Summary

This contribution discusses the course of deformation analysis of the eastern edge of the High Tatras - for this purpose, a static method with three hours of observations using each point designed appropriately chosen points of the State spatial network in the locality. It was used a three-dimensional Helmert transformation using two methods - the ordinary least squares and total least squares method - to transform of coordinates between the different epochs of deformation of the investigation. Due to the better visualization of the displacements, coordinates of the points of both epochs of deformation investigation are transformed from the European Terrestrial Reference System 1989 (ETRS89) to the Uniform Trigonometric Cadastral Network (JTSK 03). The final section of that article is devoted to the analysis of strain and consequently draw up the illustrated maps with horizontal and vertical displacements.

Keywords: deformation analysis, three-dimensional Helmert transformation, Least Squares, Total Least Square

Introduction

The detailed geological research of the Tatras began at the beginning of the 19th century. The need to explore the Tatras closely related with the effort of geological staff to solve problems in the Western Carpathians. Researching of study area and its stability would not be possible without cooperation between geologists and surveyors. One of the most important tools in examining the stability of the territory are currently surveying measurings. Contents of the present paper is to determine the 3D displacement selected points in the eastern edge of the Tatras and strain analysis of their designated area in conjunction with the solution of the tasks of the Institute of Geosciences of the Technical University of Košice. For the purpose of deformation analysis of the eastern edge Tatras were selected and consequently focused 10 points of the State Spatial Network. Displacements at various points were determined by transformation using the method of least squares, which represents fitting points of chosen epoch to starting epoch, with minimizing the sum of squares repairs only one of them. Using method of least squares, can be encountered with the fact that the coordinates of all the studied points can be affected by errors. For this reason, the processing of geodetic measurements used the total method of least squares - Total Least Squares (TLS), developed by Golub and Van Loan, through which it is possible to track errors in all coordinates.

Implementation of terrain measurements. Selecting the processing area. Primary processing of measurements

Within the State points spatial network were stabilized and for the purpose of deformation analysis of the study area (in the eastern part of the High Tatras) were selected 10 points.The individual points were chosen so that it was best captured the character of area and to be able to express this changing in area, in the time interval between two epochs, ie possible to analyze the deformation phenomenon and its progress. These are the points SK – 2063.01 and 4919 – 9 in village Starý Smokovec, point 4915 – 4 in village Veľká Lomnica, point 2731PP – 1002 in Tatranská Javorina, point ZGZH – 578 between villages Tatranská Javorina and Podspády, point 2732SL – 1006 in village Lendak, point ZGZH – 541 in Tatranska Lomnica, point ZGZH – 564 in Ždiar, point 2732SL – 1002 in Spišské Hanušovce and point NZH – 520 in Podhorany (see Fig. 1).

Position of the points determined in the first epoch was regained from survey control point that at the request of the Institute of Geodesy, Cartography and Cadastre SR issued by geodetic bases - Institute of Geodesy and Cartography in Bratislava.

The second epoch, alone field measurements, was realized in the period of October - November 2010 with the static method with three - hour observation at each point. In measuring was used dual frequency GPS receiver Leica 1200. From selected points of the State Spatial Network was not impracticable measurement of point 4919-9, for which it is established an active GPS stations operated by the Department of Theoretical Geodesy Faculty of Civil Engineering in Bratislava . The necessary data for processing were requested from the above operator.

For the purpose of deformation analysis of the High Tatras was in the processed results of geodetic measurements selected European Terrestrial Reference System 1989 – ETRS8Resolution No. 1 Technical - Steering Group of the European Reference Frame adopted on the meeting held in 1990 in Florence ETRS89 defined as a system which is identified with the International Terrestrial Reference System (ITRS) at epoch 1989.0, and which is fixed to the

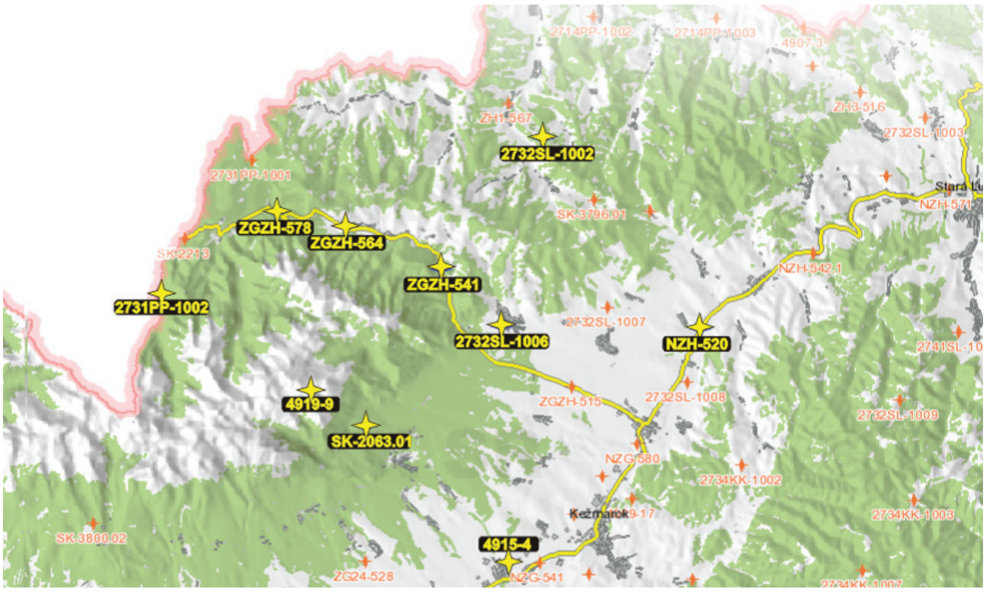

Fig. 1 Choosing of appropriate points of the State spatial network in the High Tatras [15]

Rys. 1 Wybrane punkty państwowej sieci przestrzennej w Wysokich Tatrach [15]

Fig. 2 The points SK – 2063.01and 4919-9

Rys. 2 Punkty SK - 2063.01 i 4919-9

stable part of the Eurasian Tectonic Plate. ETRS89 coordinates can be expressed either as a orthogonal Cartesian coordinates X, Y, Z or ellipsoidal (geodetic) coordinates B, L, H, where „B“ is ellipsoidal (geodetic) width, „L“, is ellipsoidal (geodetic) longitude and „H“ is ellipsoidal (geodetic) height. These are based on the ellipsoid of the Geodetic Reference System 1980 with the Greenwich Prime Meridian Position of points representing 1st epoch of deformation investigation is taken from geodetic data point in the form of ellipsoidal (geodetic) coordinates of B, L, H system ETRS89 [8],[11]. These relations have been using under rotational ellipsoid geometry and known geometric constants of the ellipsoid GRS 80 transformed to cartesian coordinates. The data measured by static method with three-hour the observation at each point were processed by means postprocessing software LEICA Geo Office. The result of processing are the cartesian coordinates X, Y and Z measuring points in a coordinate system ETRS89 (see Tab. 1).

By the transformation of coordinates points from 2nd epoch focus to points of 1st epoch was used spatial Helmert transformation. Required transformation parameters were calculated using the classical (least square method - LSM) and total least square method.

Three-dimensional Helmert transformation of coordinates using an alternative approach to LSM

Helmert transformation (see Figure 3.), has the general form [1], [9]

$$\begin{bmatrix} X_T \\ Y_T \\ Z_T \end{bmatrix}_i = \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} + \begin{bmatrix} X_S \\ Y_S \\ Z_S \end{bmatrix} + (1+m)\begin{bmatrix} 1 & -R_z & R_y \\ R_z & 1 & -R_x \\ -R_y & R_x & 1 \end{bmatrix} \cdot \begin{bmatrix} X_i - X_S \\ Y_i - Y_S \\ Z_i - Z_S \end{bmatrix}, \qquad (1)$$

Tab. 1 Coordinates of chosen points in the first and second epoch of measuring

Tab. 1 Współrzędne wybranych punktów w pierwszym i drugim etapie pomiaru

COORDINATES OF CHOSEN POINTS						
point	1 st. epoch			2. epocha		
	x [m]	y [m]	z [m]	X [m]	Y [m]	Z [m]
SK-2063.01	3919784,5957	1446497,8447	4804532,9861	3919784,5872	1446497,8210	4804532,9894
4919-9	3920489,6104	1443472,2909	4806794,7087	3920489,6792	1443472,2600	4806794,6476
4915-4	3921386,7480	1455520,7456	4799873,4049	3921386,7071	1455520,7168	4799873,2586
ZGZH-564	3912851,0372	1442096,7233	4811238,9440	3912851,0092	1442096,6960	4811238,9179
2731PP-1002	3919071,8373	1434132,0242	4808590,2404	3919071,8259	1434132,0117	4808590,2162
2732SL-1006	3913083,7560	1451222,3134	4807980,0111	3913083,7500	1451222,3089	4807980,0186
2732SL-1002	3905514,3423	1450177,6785	4814471,0843	3905514,3407	1450177,6741	4814471,0624
NZH-520	3909024,9657	1460979,2290	4808105,5993	3909024,9395	1460979,2160	4808105,5777
ZGZH-578	3913682,4946	1438448,6787	4811535,7318	3913682,4779	1438448,6726	4811535,7066
ZGZH-541	3912280,4172	1447364,6153	4809862,6618	3912280,4453	1447364,6247	4809862,7232

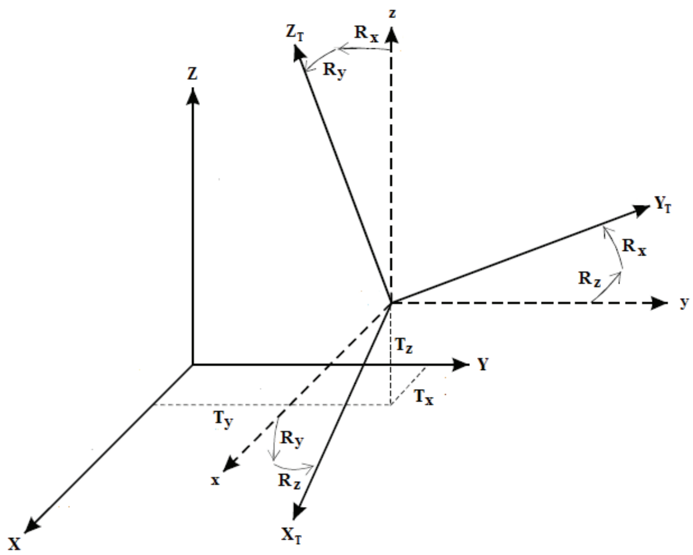

Fig. 3 3D Helmert transformation of coordinates

Rys. 3 Transformacja 3D we współrzędnych Helmerta where $(1+m)$ is scale R_x, R_y, R_z are rotational and T_x, T_y, a T_z, translational parameters $[X \quad Y \quad Z]_i^T$ are coordinates of i-th point in the original coordinate system $[X_T \quad Y_T \quad Z_T]_i^T$ are transformed coordinates of the same point and $[X_S \quad Y_S \quad Z_S]^T$ are the coordinates of the center of gravity of the area of interest in the original coordinate system.

Under identical points needed to calculate the transformation parameters have been chosen all the points of the State Spatial Network, which touches the deformation analysis Three-dimensional Helmert transformation of coordinates is in the presented paper addressed through a total least squares method.

a) Helmert transformation of coordinates using the classical least squares method

Let us have a system of linear equations, which has the following form [1]:

$$A.\theta = L \tag{2}$$

where L is a vector of observations, A full matrix of design (partial derivatives) and Θ vector of estimated parameters.

Least squares method searching for the best compilation of vector Θ providing, that the vector of observations L is loaded with random errors, while the matrix of design A is not random errors of subject. Model of Least squares method then takes the form of:

$$A.\theta - (L - E_L) = 0, \tag{3}$$

where E_L is matrix of erros vector of observations L. Simple mathematical adjustments and adding weights Q to each observation we get the formula to determine of estimated parameters:

$$\theta = \left(A^T.Q^{-1}.A\right)^{-1}.A^T.Q^{-1}.L. \tag{4}$$

In the case of Helmert transformation is the vector of estimated parameters Θ vector of transformation parameters:

$$\theta = \begin{bmatrix} T_x \\ T_y \\ T_z \\ 1+m \\ R_x \\ R_y \\ R_z \end{bmatrix}_{(7\times 1)} \tag{5}$$

Matrix of design A consists of two submatrix A_1 a A_2 [1]:

$$A = [A_1 \quad A_2], \tag{6}$$

$$Where \ A_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \vdots & \vdots & \vdots \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{(3p\times 3)}$$

$$A_2 = \begin{bmatrix} X_1 - X_S & 0 & -(Z_1 - Z_S) & Y_1 - Y_S \\ Y_1 - Y_S & Z_1 - Z_S & 0 & -(X_1 - X_S) \\ Z_1 - Z_S & -(Y_1 - Y_S) & X_1 - X_S & 0 \\ \vdots & \vdots & \vdots & \vdots \\ X_p - X_S & 0 & -(Z_p - Z_S) & Y_p - Y_S \\ Y_p - Y_S & Z_p - Z_S & 0 & -(X_p - X_S) \\ Z_p - Z_S & -(Y_p - Y_S) & X_p - X_S & 0 \end{bmatrix}_{(3p\times 4)}$$

p represents the number of identical points selected for the coordinates transformation.

Vector of observations L is expressed as:

$$L = \begin{bmatrix} x_1 - X_1 & y_1 - Y_1 & z_1 - Z_1 & \cdots & x_p - X_p & y_p - Y_p & z_p - Z_p \end{bmatrix}^T. \tag{7}$$

b) Helmert transformation of coordinates using the total least squares method

Total least squares method (TLSM) is an alternative to the classic least squares method, when the observations, and members of the matrix of design are burdened with random errors. The mathematical model of this method is expressed as follows [1]:

$$(A - E_A).\theta = (L - E_L), \tag{8}$$

where E_L is vector of errors of observations and E_A is errors matrix of matrix of design.

General solution consists of the following [1]:

a) QR factorization (decomposition of the matrix on multiply of orthogonal and upper triangular matrix) expanded matrix $D.[A_{1.} A_2 \ L]$ is calculated as:

$$Q^T.D.[A_1; A_2; L] = \begin{bmatrix} R_{11} & R_{12} & R_{1b} \\ 0 & R_{22} & R_{2b} \end{bmatrix}. \tag{9}$$

where D is diagonal weighting matrix observations dimension (n x n);

b) With a second row from the relation (9), solution

$d\hat{k}_2$ for the reduced system

$$[R_{22}; R_{2b}].C.\left(C^{-1}.\begin{bmatrix} d\hat{k}_2 \\ -L \end{bmatrix}\right) \approx 0$$

is obtained using singulare value decomposition for $[R_{22} \quad R_{2b}].C = U.\Sigma.V^T$ and $d\hat{k}_2$ is estimated as

$$d\hat{k}_2 = -\frac{1}{c_{m_2+1}.v_{m_2+1,m_2+1}}.C_{1\ldots m_2}.\left[v_{1,m_2+1}, v_{2,m_2+1}, \ldots v_{m_2,m_2+1}\right]^T, \tag{10}$$

where C is diagonal weighting matrix, which shows the relative precision observables with respect to members of the matrix of design in columns of submatrix A_2, which constitutes part of the matrix A loaded with errors, $C_{1\ldots m2} = diag(c_1,c_2,\ldots c_{m2})$ are diagonal elements of the matrix C in the first m_2 row (or in column), c_{m2+1} is lowest stored diagonal matrix element C, which is the weight of vector observations with respect to columns submatrix A_2.

c) Parameters $d\hat{k}_2$ are determined using the the first row of the formula (9) using the backward substitution parameters $d\hat{k}_2$ estimated in the second step as:

$$d\hat{k}_1 = R_{11}^{-1}.\left(R_{1b} - R_{12}.d\hat{k}_2\right). \tag{11}$$

By Helmert transformation in the shape and size of the vector and observations L and matrix of design does not change, unknown transformation parameters are determined in the form:

$$d\hat{k}_1 = \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix}, \quad d\hat{k}_2 = \begin{bmatrix} 1+m \\ R_x \\ R_y \\ R_z \end{bmatrix}. \tag{12}$$

Weighting matrix D and C are defined as follows:

$$diag(D) = \left[\Sigma_{x_1x_1} \quad \Sigma_{y_1y_1} \quad \Sigma_{z_1z_1} \quad \cdots \quad \Sigma_{x_px_p} \quad \Sigma_{y_py_p} \quad \Sigma_{z_pz_p}\right] \tag{13}$$

$$C = \begin{bmatrix} \frac{tr(\Sigma_{XYZ})}{tr(D)} & 0 & 0 & 0 & 0 \\ 0 & \frac{tr(\Sigma_{YZ})}{tr(D)} & 0 & 0 & 0 \\ 0 & 0 & \frac{tr(\Sigma_{XZ})}{tr(D)} & 0 & 0 \\ 0 & 0 & 0 & \frac{tr(\Sigma_{XY})}{tr(D)} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \tag{14}$$

where tr() je „trace" - trace of matrix, members of matrix D are weights of individual observations (coordinate of system xyz), $\sum_{XYZ}$, $\sum_{YZ}$, $\sum_{XZ}$, $\sum_{XY}$ are submatrix weights of identical points in the transformed coordinate system (system XYZ).

Transformation parameters of three-dimensional Helmert transformation determined by the transformation least squares method (LS) and total least squares method (TLS) are listed in Table 2, Table 3 contains the transformed coordinates of 2nd epoch measurements.

Identification congruent test

In order to be confirmed with a certain probability, that changes in the point position in the direction of the coordinate axes presented the coordinate differences points between 1st and already transformed 2nd epoch is a real move the points and not just the accumulation of errors, these values were statistically tested.

In order to evaluate the stability of individual points was performed identification test of kongruentnosti, in which by comparing the relevant of the test statistic with the critical value we can find out, whether movement of a particular point is, or not is statistically significant.

Let the deformation vector for i-th point of deformation network has the form [1], [6],[14],[13]

$$d_i = \begin{bmatrix} x_{Ti} - x_i \\ y_{Ti} - y_i \\ z_{Ti} - z_i \end{bmatrix}_{(3,1)} = \begin{bmatrix} dx_i \\ dy_i \\ dz_i \end{bmatrix}_{(3,1)} \tag{15}$$

The selected level of significance: α=0,05 (16)

The null hypothesis: $H_0 : d_i = 0$ (17)

Alternative hypothesis: $H_A : d_i \neq 0$ (18)

The test of statistic: $T_i = \frac{d_i^T.Q_{dd_i}^{-1}.d_i}{3.s_0^2}$ (19)

Critical value: $T_{krit} \approx F_\alpha\left(3,(n-k)\right)$ (20)

If $T_i < T_{krit}$ - accepts the null hypothesis H_0 (17), movement of the point is not statistically significant, point has a stable character, if $T_i < T_{krit}$ - null hypothesis H_0 (17) is rejected, accepts the alternative hypothesis H_A (18), movement of the point is statistically significant. Local congruent test was performed twice (the transformed coordinates of 2nd epochs determined by methods LS and TLS). Numeric representation of the critical values of the of the test statistic of each point indicates Table 4.

From the results of the congruent test (see Tab. 4) It is evident that all the points, the processing of which was used LS metóda have remained stable, unchanged, in determining points TLS method, test

Tab. 2 The transformation parameters determined by the LS and the TLS

Tab. 2 Parametry transformacji określonych przez LS i TLS

Parameter	T_x [m]	T_y [m]	T_z [m]	1+m	R_x [´´]	R_y [´´]	R_z [´´]
LS method	0,000478	0,013754	0,019039	0,999997	-0,711614	1,668682	-0,084569
TLS method	-0,017298	0,012655	-0,017320	0,999996	-1,544923	1,782128	0,474409

Tab. 3 Transformed coordinates of the second epoch determined by the LS and the TLS

Rys. 5. Krzywe płynięcia wg modelu Binghama, zawiesin sporządzonych z popiołu K-1 bez dodatku cementu

TRANSFORMED COORDINATES OF 2 nd EPOCH						
	LS method			***TLS method***		
point	$X_{T(LS)}$ [m]	$Y_{T(LS)}$ [m]	$Z_{T(LS)}$ [m]	$X_{T(TLS)}$ [m]	$Y_{T(TLS)}$ [m]	$Z_{T(TLS)}$ [m]
SK-2063.01	*3919784,6053*	*1446497,8511*	*4804533,0574*	*3919784,5801*	*1446497,8522*	*4804533,0280*
4919-9	*3920489,6092*	*1443472,3213*	*4806794,7666*	*3920489,5734*	*1443472,3164*	*4806794,7216*
4915-4	*3921386,7551*	*1455520,7404*	*4799873,3827*	*3921386,7544*	*1455520,7415*	*4799873,3980*
ZGZH-564	*3912850,9927*	*1442096,7114*	*4811238,8974*	*3912850,9630*	*1442096,7114*	*4811238,8355*
2731PP-1002	*3919071,8181*	*1434132,0593*	*4808590,2254*	*3919071,7583*	*1434132,0660*	*4808590,1390*
2732SL-1006	*3913083,7555*	*1451222,3122*	*4807980,0398*	*3913083,7520*	*1451222,3100*	*4807980,0202*
2732SL-1002	*3905514,3136*	*1450177,6546*	*4814471,0021*	*3905514,3159*	*1450177,6484*	*4814470,9635*
NZH-520	*3909024,9505*	*1460979,1921*	*4808105,5994*	*3909024,9799*	*1460979,1847*	*4808105,6167*
ZGZH-578	*3913682,4583*	*1438448,6968*	*4811535,6795*	*3913682,4173*	*1438448,6992*	*4811535,6028*
ZGZH-541	*3912280,4392*	*1447364,6311*	*4809862,7198*	*3912280,4255*	*1447364,6297*	*4809862,6811*

Tab. 4 Identification congruent test

Rys. 5. Krzywe płynięcia wg modelu Binghama, zawiesin sporządzonych z popiołu K-1 bez dodatku cementu

point	T_i		T_{krit}	**Conclusion**	
	LS	**TLS**		**LS**	**TLS**
SK-2063.01	1,3032	0,7519	3,0280	-	-
4919-9	0,0079	0,0063		-	-
4915-4	1,1763	0,2503		-	-
ZGZH-564	1,0290	4,7113		-	/
2731PP-1002	0,7151	3,5379		-	/
2732SL-1006	1,7059	0,2629		-	-
2732SL-1002	1,2770	2,8468		-	- /
NZH-520	0,0067	0,0208		-	-
ZGZH-578	0,4336	2,9101		-	- /
ZGZH-541	0,0119	0,0038		-	-

Tab. 5 UTCN coordinates of the first epoch of measuring

Tab. 5 UTCN współrzędne pierwszego etapu pomiaru

point	X_{JTSK} [m]	Y_{JTSK} [m]	**H [m]**
SK-2063.01	1184729,3701	333407,6654	1173,9773
4919-9	1182778,7871	336377,6237	2634,0099
4915-4	1191736,2305	325903,6769	681,0506
ZGZH-564	1174187,6735	334508,4799	1009,7368
2731PP-1002	1177666,4807	344357,6033	1021,6185
2732SL-1006	1179370,8341	326324,0045	749,1506
2732SL-1002	1169597,1210	324096,1852	801,9017
NZH-520	1179594,5378	315760,9357	583,9043
ZGZH-578	1173407,7159	338178,1368	921,5425
ZGZH-541	1176365,1931	338178,1368	808,5393

indicates a change point position ZGZH – 564 and 2731PP – 1002. Movement of points 2732SL – 1002 and ZGZH – 578 is questionable – value of the statistic test are coming to an critical value. It is noted that the difference in results between the estimates of displacement LS and TLS method comes the maximum extent from the adding weights in TLS method, whereby it has become more sensitive. In the case where the two sets of coordinates were weighed an identity matrix, the results were closer to each other. For better visualization of displacement were before you start the strain analysis coordinates of the points 1 st and 2 nd epoch transformed with LEICA Geo Office software from the system ETRS89 into the system S-JTSK [9], [10] in the valid implementation JTSK03 (see Table 5, Table 6).

Figure 4 and Figure 5 show the average annual and total displacements of points, whose the second epoch coordinates were determined using LS and TLS methods. Time span of each measurement points is 10 (4919-9), 9 (4915-4) and 8 (SK-2063.01, ZGZH-564, 2731PP-1002, 2732SL-1006, 2732SL-1002, NZH-520, ZGZH-578 a ZGZH-541) years. Diversity of The offset is caused by the addition of weights in TLS method, thereby it is more sensitive. Compared with the displayed coordinate system the displacements are drawn as 100 000 times magnified.

Strain deformation analysis of points

Each solid can influence action of endogenous and exogenous forces undergo changes in its shape and position, made deformation. These changes can occur either by sequentially or simultaneously.

Deformation of the solid is determined by the the transfer of particles (points) of the solid. Let us vector of particles (point) p in three-dimensional cartesian coordinate system xyz before deformation - r_p and after deformation - r'_p. Vector r'_p can be expressed as follows [3],[2],[15]:

$$r'_p = r_p(x_p, y_p, z_p, t). \tag{21}$$

As the end position of point depends on the input of coordinates and time, vector of position changes d_p particle p write as:

$$d_p = r'_p - r_p = d_p(x_p, y_p, z_p, t). \tag{22}$$

Formula (22) can be decomposed into three parts: translation and rotation of the solid as a whole and its own deformation, so that the vector r'_p can be described as:

$$r'_p = t + R.r_p + \tilde{d}_p, \tag{23}$$

where $t^T = (t_x, t_y, t_z)$ is the vector of solid translation, R rotation matrix and $\tilde{d}_p$ presents own deformation. For describing deformation in the surrounding area of point is used strain analysis. If the displacements are very small in comparison to the dimensions of the solid, then results in nonnumeric translational deformation tensor ε have the form [3]

$$\varepsilon = \begin{pmatrix} \frac{\partial dx}{\partial x} & \frac{\partial dx}{\partial y} & \frac{\partial dx}{\partial z} \\ \frac{\partial dy}{\partial x} & \frac{\partial dy}{\partial y} & \frac{\partial dy}{\partial z} \\ \frac{\partial dz}{\partial x} & \frac{\partial dz}{\partial y} & \frac{\partial dz}{\partial z} \end{pmatrix} = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{pmatrix}, \tag{24}$$

where dx, dy and dz are three components of vector displacement in the direction of x, y and z. Deformation tensor ε is asymmetrical and for this reason can be decomposed into symmetric and non-symmetric

Tab. 6 UTCN coordinates of the second epoch of measuring determined by the LS and the TLS

Tab. 6 UTCN współrzędne drugiego typu pomiaru określone przez LS i TLS

	LS method			TLS method		
point	$X_{JTSK\,(LSM)}$ [m]	$Y_{JTSK\,(LSM)}$ [m]	$H_{(LSM)}$ [m]	$X_{JTSK\,(TLS)}$ [m]	$Y_{JTSK\,(TLS)}$ [m]	$H_{(TLS)}$ [m]
SK-2063.01	1184729,3323	333407,6604	1174,0386	1184729,3345	333407,6508	1174,0011
4919-9	1182778,7584	336377,5930	2634,0598	1182778,7615	336377,5854	2634,0027
4915-4	1191736,2482	325903,6853	681,0369	1191736,2381	325903,6835	681,0483
ZGZH-564	1174187,6695	334508,4754	1009,6714	1174187,6894	334508,4663	1009,6064
2731PP-1002	1177666,4884	344357,5642	1021,6032	1177666,5057	344357,5383	1021,5026
2732SL-1006	1179370,8148	326324,0043	749,1717	1179370,8244	326324,0058	749,1542
2732SL-1002	1169597,1473	324096,1991	801,8164	1169597,1720	324096,2072	801,7871
NZH-520	1179594,5156	315760,9637	583,8866	1179594,5222	315760,9813	583,9160
ZGZH-578	1173407,7307	338178,1081	921,4847	1173407,7532	338178,0931	921,4020
ZGZH-541	1176365,1755	329490,7206	808,6003	1176365,1908	329490,7180	808,5622

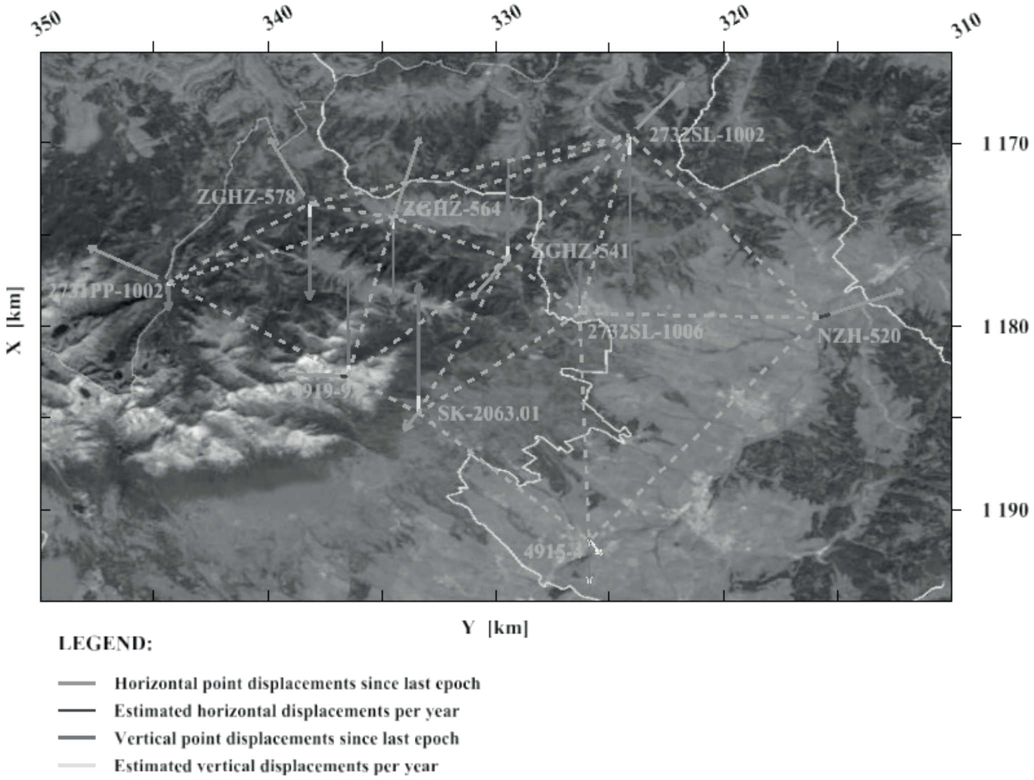

Fig. 4 Average annual and total 3D displacements of points - LS method

Rys. 4 Średnie roczne i całkowite przemieszczenia 3D punktów - Metoda LS

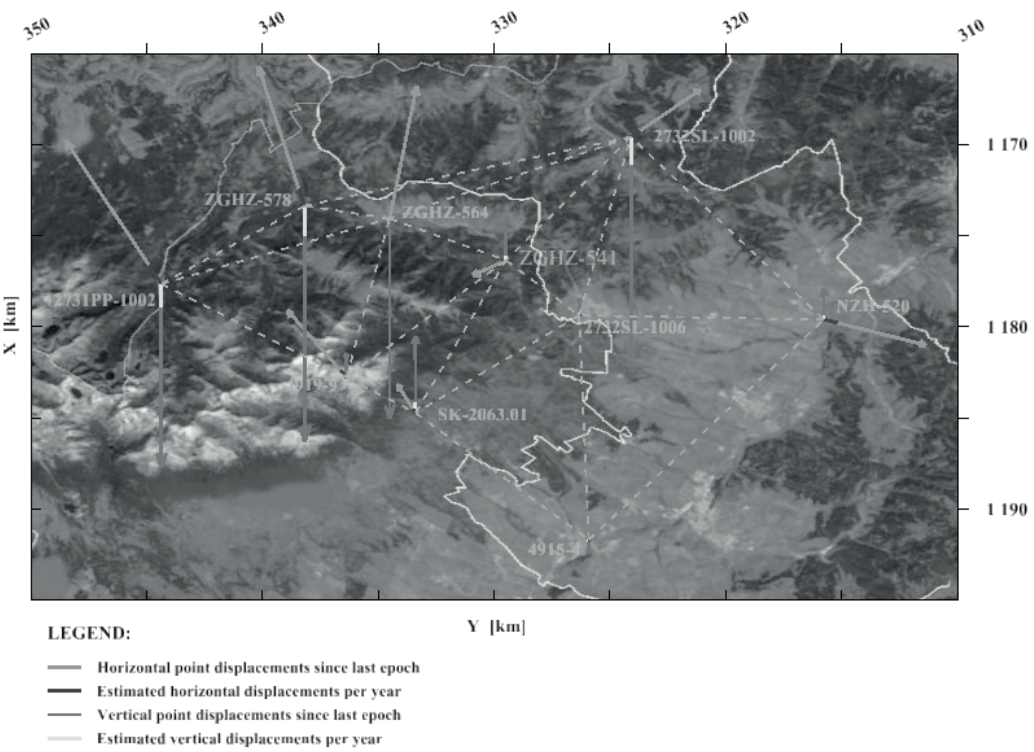

Fig. 5 Average annual and total 3D displacements of points - TLS method

Rys. 5 Średnie roczne i całkowite przemieszczenia 3D punktów - metoda TLS part of the according formula:

$$\varepsilon = \frac{1}{2}.\left(\varepsilon + \varepsilon^T\right) + \frac{1}{2}.\left(\varepsilon - \varepsilon^T\right) = e_j + \omega_j \tag{25}$$

where
$$e_{ij} = \frac{1}{2}.\left(\varepsilon_{ij} + \varepsilon_{ji}\right)$$
$$\omega_{ij} = \frac{1}{2}.\left(\varepsilon_{ij} - \varepsilon_{ji}\right) \quad \text{for i, j = x, y, z} \tag{26}$$

where e_{ij} is called the strain tensor and ω_{ij} represents the rotation diagonal elements e_{ij} marked dilatation for a particular direction, called extensible strain and off-diagonal elements characterized by displacement angle between the input lines, called also shear strain.

Figure 6 represents the geometric interpretation of the strain components, where the original solid (cube with sides of the same length) is drawn by dashed line and deformed form by soft line. As displacements vector ^{d} depends on the location and time, and also the elements of strain tensor and rotation are generally dependent on the position and time. If partial derivative is with the respect to time, we can get the intensity of strain e_{ij} components and intensity of the rotational component ω_{ij} . On the other side, the rotary component ω_{ij} can be divided into two parts: not dependent on the position ω_{ij}^o and and dependent on the position ω_{ij}' . As different parts of the deformed solid were

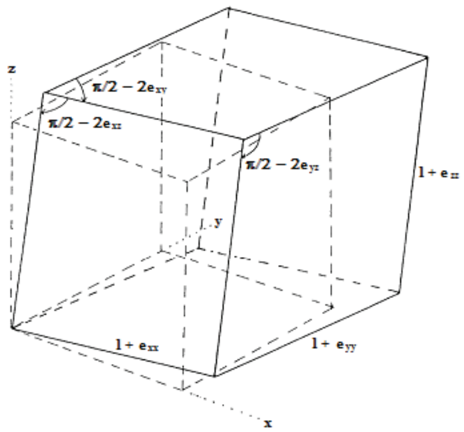

Fig. 6 Geometric interpretation strain elements

Rys. 6 Interpretacja geometrycznych elementów odkształceń

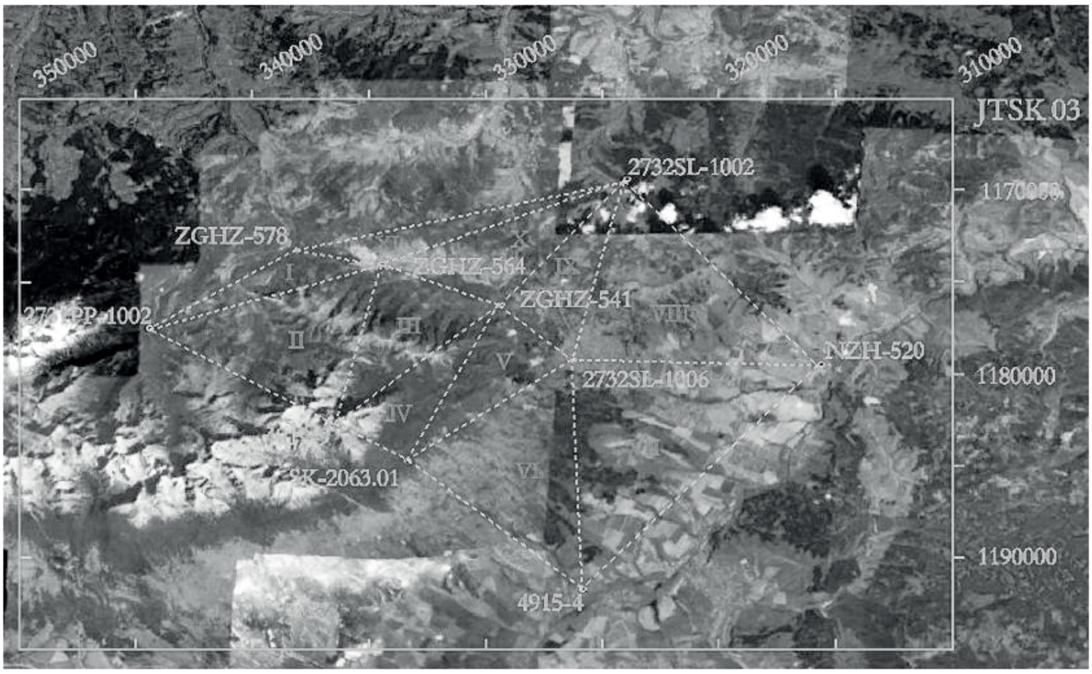

Fig. 7 Regional parts deciding of the location of the High Tatras for requisities strain analysis

Rys. 7 Elementy regionu decydujące o lokalizacji w Tatrach Wysokich wymaganych szczepów do analizy exposed to the same deformationindependent of their position, deformation is homogeneous.

Strain analysis of points the eastern edge of the High Tatras in the plane XY

Deformation strain analysis can be used not only for analysis of 2D (or 3D) [12] movement changes the whole monitored area but also its parts separately, and it, that, the results of these analyzes are more credible. The observed location of the High Tatras by creating appropriate triples of points (see Table 7) divided into 11 smaller parts of the territory triangular shape [4],[5],[7] (see Figure 7).

Positional transfer i-th point for the monitored period, can be express with the formula [2],[3],[5],[8]

$$d_i = \begin{bmatrix} dx \\ dy \end{bmatrix}_i = \left(\begin{bmatrix} e_{xx} & e_{xy} \\ e_{xy} & e_{yy} \end{bmatrix} + \begin{bmatrix} 0 & \omega_{xy} \\ -\omega_{xy} & 0 \end{bmatrix} \right) . \begin{bmatrix} X \\ Y \end{bmatrix}_i + \begin{bmatrix} T_x \\ T_y \end{bmatrix}, \qquad (27)$$

where $\begin{bmatrix} X & Y \end{bmatrix}_i^T$ are coordinates of point in 2nd epoch of research and Θ represents the vector of deformation parameters expressed in the form:

$$\theta = \begin{bmatrix} e_{xx} & e_{xy} & e_{yy} & \omega_{xy} & T_x & T_y \end{bmatrix}^T . \qquad (28)$$

Vector d_i characterizing the point displacement and its surroundings in the direction of the coordinate axes is defined as:

$$d_i = H_i . \theta, \qquad (29)$$

where H_i is coefficients matrix containing the coordinates of 2nd epoch:

$$H_i = \begin{bmatrix} X & Y & 0 & Y & 1 & 0 \\ 0 & X & Y & -X & 0 & 1 \end{bmatrix} . \qquad (30)$$

By extending the matrix H and vector d three points can express also the deformation parameters relate on the territory of the triangular shape. Simple mathematical modifications of formula (27) using the least squares method and the addition of weights each coordinate difference get the formula to determine the deformation parameters:

$$\theta = \left(H^T . Q^{-1} . H \right)^{-1} . H^T . Q^{-1} . d . \qquad (31)$$

Tab. 7 Creating regional part of the location for the purposes of strain analysis

Tab. 7 Tworzenie części regionalnej lokalizacji dla celów analizy naprężeń

Δ	I	II	III	IV	V	VI
point	ZGZG-578 2731PP-1002 ZGZH-564	2731PP-1002 ZGZH-564 4919-9	ZGZH-564 4919-9 ZGZH-541	4919-9 ZGZH-541 SK-2063.01	ZGZH-541 SK-2063.01 2732SL-1006	SK-2063.01 2732SL-1006 4915-4
Δ	**VII**	**VIII**	**IX**	**X**	**XI**	
point	2732SL-1006 4915-4 NZH-520	2732SL-1006 NZH-520 2732SL-1002	2732SL-1006 2732SL-1002 ZGZH-541	2732SL-1002 ZGZH-541 ZGZH-564	2732SL-1002 ZGZH-564 ZGZG-578	

Tab. 8 Deformation parameters of strain analysis – LS and TLS

Tab . 8 Deformacja parametrów analizy naprężeń - LS i TLS

	LS method						TLS method					
Δ	e_{xx} [μstrain]	e_{xy} [μstrain]	e_{yy} [μstrain]	ω_{xy} [´´]	Tx [m]	Ty [m]	e_{xx} [μstrain]	e_{xy} [μstrain]	e_{yy} [μstrain]	ω_{xy} [´´]	Tx [m]	Ty [m]
I	-6,956	4,546	-5,437	-0,1859	0,0062	-0,0241	-8,675	4,631	-7,082	-0,1326	0,0261	-0,0408
II	-3,394	-0,044	-2,639	0,5015	-0,0083	-0,0248	-5,450	0,482	-4,553	0,4882	0.0051	-0,0390
III	-3,166	-0,798	-0,535	0,4403	-0,0168	-0,0145	-5,135	-0,559	-1,650	0,4038	-0,0040	-0.0209
IV	-2,944	2,261	-6,426	-0,2332	-0,0280	-0,0146	-4,251	1,543	-6,331	-0,1996	-0,0211	-0,0212
V	-1,846	-0,066	-1,498	-0,2371	-0,0249	-0,0045	-3,514	-0,037	-2,944	-0,1983	-0,0159	-0.0080
VI	2,831	-2,049	-1,174	-0,5578	-0,0131	0,0011	1,243	-2,126	-2,504	-0,5095	-0,0126	-0,0022
VII	3,004	0,472	-2,657	-0,0275	-0,0079	0,0121	1,419	0,437	-4,188	0,0312	-0,0059	0,0178
VIII	-4,705	-0,328	-2,687	0,1036	-0,0051	0,0139	-6,307	-0,366	-4,218	0,1631	0,0086	0,0230
IX	-3,936	-1,956	-3,203	-0,2564	-0,0035	0,0018	-5,544	-1,975	-4,794	-0,1959	0,0130	0,0042
X	-6,425	-1,470	-0,505	0,2872	0,0016	0,0004	-7,999	-1,557	-1,977	0,3533	0,0215	-0,0008
XI	-12,294	4,949	-5,024	-0,5031	0,0124	-0,0064	-14,084	5,079	-6,647	-0,4623	0,0347	-0,0118

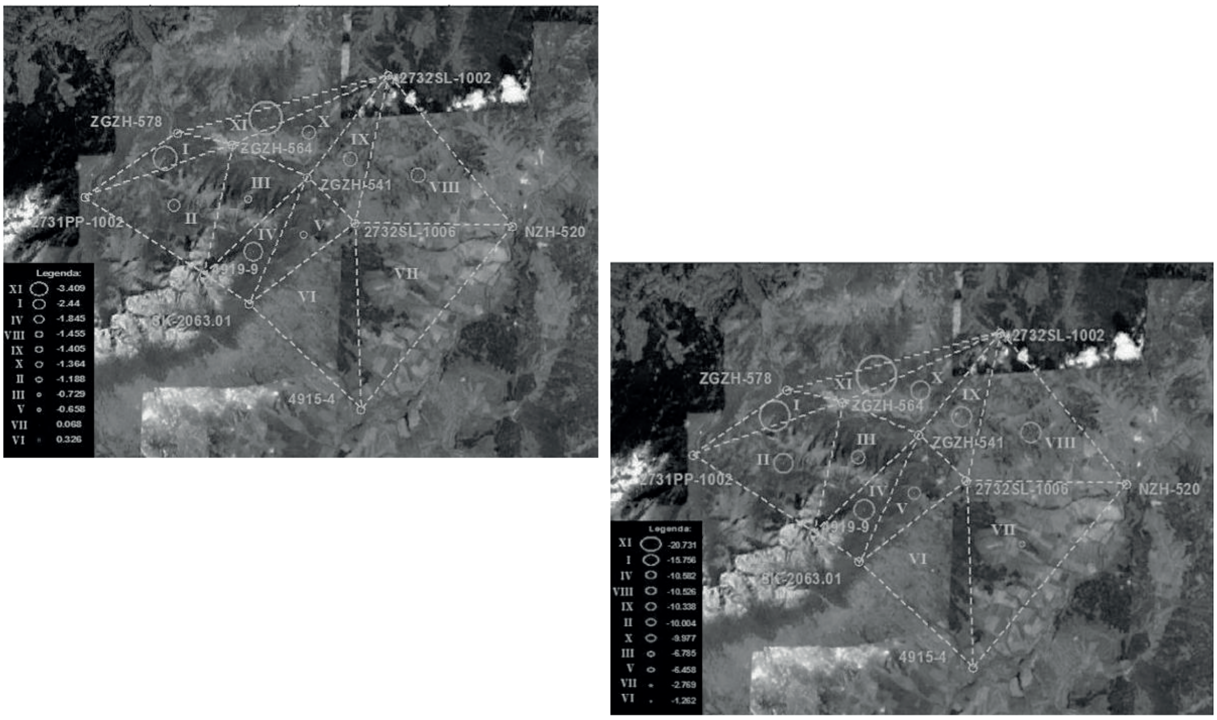

Fig. 8 Relative planary deformations shown by circular lines – LS and TLS

Rys. 8 Względne odkształcenia plenarne przedstawione przez okrągłych liniach - LS i TLS

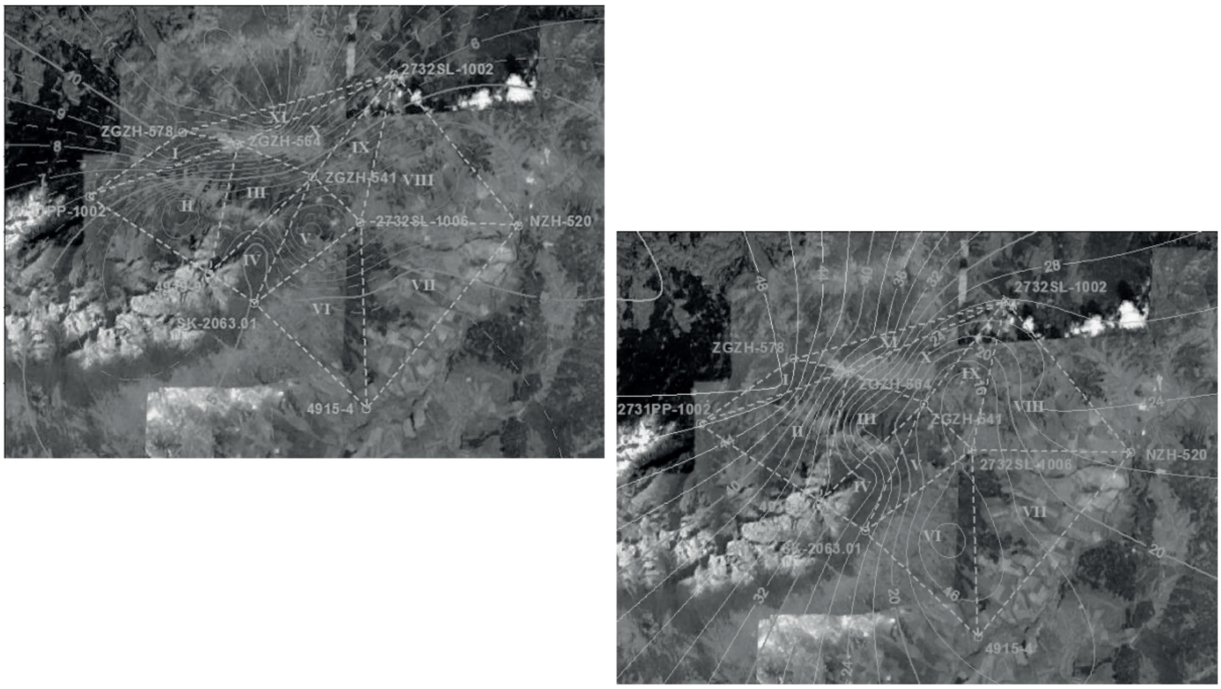

Fig. 9 Relative shear deformations shown by isolines – LS and TLS

Rys. 9 Względne odkształcenia ścinające na obrazie izolinii - LS i TLS

Tab. 9 The values of relative planary and shear deformation and horizontal displacements

Tab . 9 Wartości względne odkształceń planarnych i ścinania oraz przemieszczeń poziomych

Δ	**Δ [µstrain]**		**γ [µstrain]**		**Δh [mm]**		**Δ**	**Δ [µstrain]**		**γ [µstrain]**		**Δh [mm]**	
	LS	TLS	LS	TLS	LS	TLS		LS	TLS	LS	TLS	LS	TLS
I	-12,393	-15,756	9,218	9,398	24,876	48,388	**VII**	0,347	-2,769	5,739	5,675	14,441	18,784
II	-6,033	-10,004	0,761	1,317	26,131	39,299	**VIII**	-7,393	-10,526	2,122	2,213	14,795	24,512
III	-3,701	-6,785	3,077	3,660	22,145	21,279	**IX**	-7,139	-10,338	3,980	4,021	3,981	13,651
IV	-9,370	-10,582	5,707	3,722	31,623	29,981	**X**	-6,930	-9,977	6,610	6,780	1,617	21,548
V	-3,343	-6,458	0,372	0,575	25,297	17,784	**XI**	-17,319	-20,731	12,281	12,590	13,940	36,672
VI	1,657	-1,262	5,729	5,668	13,177	12,764							

Expansion or squeezing of territorial unit between two epochs reflecting the relative surface deformation area or dilation Δ in units µstrain:

$$\Delta = e_{xx} + e_{yy}, \tag{32}$$

which we can view as a circle with a radius Δ.
Another form of strain analysis interpretation is relative shear deformation γ:

$$\gamma = \sqrt{\left(e_{xx} - e_{yy}\right)^2 + \left(2.e_{xy}\right)^2}, \tag{33}$$

expressing the relative changes of actual angles changes for the period. Graphically it can be expressed by using isolines with values in µstrain.

Layout of horizontal displacements Δ*h* view isolines, which are determined according:

$$\Delta h = \sqrt{T_x^2 + T_y^2}. \tag{34}$$

Relative changes of monitored area are illustrated by circles (see Figure 8.). Circle displayed in orange and blue color represent the squeezing area, red color represent circle turn the expansion area.

Isolines of relative shear deformation (see Figure 9.) are illustrated with an interval of 1 mm, whichever, that, 1 mm = 1 µstrain. The value related to center of gravity of the area are shown in Table 9.

Conclusion

Contents of the present paper was to identify and analyze 3D of displacement selected points of ŠPS eastern edge of the High Tatras the time interval between two epochs of studies. Position of the points determined in the first epoch was taken from over the data of geodetic point, the second epoch focused dual frequency Leica GPS 1200 from October - November 2010 three-hour static method with the observation at each point. The measured data were processed by using postprocessing software LEICA Geo Office.

For the coordinates transformation from the of points 2nd epoch of studies into the points 1 st epoch was used spatial Helmert transformation. The required transformation parameters were calculated by using the classical least squares method (LS) and total (TLS) least squares method. The difference of the results were a greater extent caused the gradual of weights addition. While the LS considered only with the vector observables loaded with random errors, which tend to minimize and is expected, that members of the matrix design are not loaded with errors, TLS method consider for incorrect also matrix of design, thus in comparison with the previous method becomes more sensitive.

For a comprehensive assessment of the stability or instability of the observed location has been chosen strain analysis as a group of deformation analysis.

Strain analysis was performed twice - for set of points, to the processing was applied classical and total least squares method. Obtained horizontal displacements, relative planary and relative shear deformations of area and maximal and minimal tensile deformations are presented as circles, and isolines on the underlying orthophotomaps.

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Streszczenie

W artykule przedstawiono wyniki badań nad możliwością wykorzystania popiołów lotnych w technologiach zawiesinowych stosowanych w górnictwie podziemnym. Zawiesiny sporządzono z dwóch popiołów pochodzących z różnych instalacji, spalających komunalne osady ściekowe w kotłach fluidalnych. Właściwości zawiesin jak i kierunek ich zastosowania określono zgodnie z Polską Normą PN-G-11011 1998. Sporządzone zawiesiny nie spełniały wymagań dotyczących zastosowania w podsadzce zestalanej, natomiast w zależności od ich składu mogą być stosowane do izolacji i doszczelniania zrobów zawałowych.

Słowa kluczowe: technologie zawiesinowe, popioły lotne, spalanie komunalnych osadów ściekowych