# Assessment of Coal Mineral Matter Liberation Efficiency Index

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*Summary*

*The main tool allowing to forecast effects of gravitational beneficiation of hard coals is density analysis. The obtained results of such analysis should show real minerals liberation level in connection with their physical properties. In purpose of calculating mineral phase liberation from hard coal the size and density analysis was performed for raw hard coal which was feed directed to mineral processing. For each obtained size-density fraction its yield and ash contents were determined.*

*The Hall's separation curve was used in the paper to evaluate liberation efficiency of mineral phase from hard coal. The advantage of this curve is the fact that it is possible to describe it mathematically by means of hyperbolic equation. It was stated that if particle size is lower the liberation level of mineral substance grows.*

*Keywords: liberation efficiency. hard coal. particle size distribution. particle density distribution*

## 1. Introduction

No matter what method and way of useful component liberation from gangue is applied. the raw material beneficiation requires earlier application of many preparation processes such as comminution and classification in particular. To achieve a positive effect of this operation. the useful components must be sufficiently liberated from gangue. The material contained in deposit is the so-called continuous phase in which the useful materials particles are dissipated. In the case of hard coal, the continuous phase is coal substance and the dissipated one are particles of gangue. The incrustations may be of various form, shape and size (Niedoba, 2009). Due to significant differences in the physical properties of the feed transported to the beneficiation process. which means high differentiation of useful component and gangue properties, the issue of the beneficiation itself is connected directly with particles liberation allowing for a better or worse process efficiency (Miller et al. 2009).

The separation parameter is the physical or geometrical property, which provides a basis for the separation of particles into subsets without overlap and of various values of this property. In industrial separation processes the most often occurring separation leads to obtaining two or three products. However, this separation is not precise because of the phenomenon of particles dissipation to other separation products. In the case of the beneficiation processes, the differentiation of component amounts is usually connected with a different value of the separation parameter (Blaschke and Nycz, 2007; Brożek and Surowiak, 2006).

The simple and complex properties can be distinguished. The simple ones include such parameters as density, magnetic susceptibility or particle size. What is characteristic of them is the fact that only the value of one property decides about their affiliation to a specific subset. The value of the property is conditioned only by particle chemical composition or way of mineral particle creation. The distributions of these properties in the sample are one-dimensional.

The distribution function of physical properties such as density or magnetic susceptibility is obtained from physical particle model, i.e. a dispersive model (Brożek, 1995a; 1995b; Wei and Gey, 1999). In this model the continuous phase (gangue or organic coal substance) is warp in which the incrustations of useful components or gangue are suspended, creating the dissipated phase. For the two-component raw material the particle density $\rho$ is given by the following equation:

$$\rho = \rho_k \lambda + (1-\lambda)\rho_0 \qquad (1)$$

where:

- $\rho_k$ – dissipated phase incrustations density,
- $\rho_0$ – continuous phase density,
- $\lambda$ – volumetric contents of dissipated phase.

The physical property value of a specific raw material depends on volumetric contents of dissipated phase, which is connected with the number of incrustations of this phase in the warp of continuous phase.

Many previously elaborated models concerning the liberation of mineral particles from incrustations are geometrical ones (Bonifazi and Massacci, 1995;

Ito et al. 2009), which take the geometry of incrustations into consideration without considering the physical properties which take crucial part in the beneficiation process. The main tool which makes it possible to forecast the gravitational beneficiation results for hard coal is dense metric analysis. The results obtained in the analysis should reflect the real liberation efficiency of minerals in relation to their physical properties (Oki et al. 2004). The fractional analysis also allows for a graphical presentation of obtained results, which facilitates interpretation.

## 2. Experiment methodology

### *2.1. Hall's beneficiation curve and its approximation*

To describe the results of dense metric analysis the separation curve presenting the function $\beta = f(\varepsilon)$ can be applied. This is the so-called Halls' separation curve, where $\beta$ is the ash contents in the tailings (tailings grade) and $\varepsilon$ is the yield of ash in tailings (tailings ash recovery) (Brożek, 1984; Hall, 1971). This is the opposite form of the very well-known Halbich's beneficiation curve where $\varepsilon = f(\beta)$ (Drzymała, 2001 a, b; Halbich, 1934). According to Kelly and Spottiswood (1982). the best beneficiation point on this curve can be found by way of beneficiation selectivity curves defined as follows:

$$f = \frac{\varepsilon\beta}{100} \tag{2}$$

The mathematical form of this curve suggests that the curve is conic. After transformation. it can be written as follows (Brożek, 1984):

$$x = 100 - \varepsilon \tag{3}$$

$$y = 100\frac{\beta_t - \beta}{\beta_t - \alpha} \tag{4}$$

where:

$\alpha$ – ash contents in feed (feed grade),

$\beta_t$ – theoretically possible ash contents in pure tailings (theoretical tailings grade).

The formula for $y$ can be easily approximated to the hyperbola equation of the general form:

$$y = \frac{B}{x + C} - A \tag{5}$$

Where $A$, $B$, $C$ are equation parameters calculated by regressive method.

After some transformations, it occurs that:

B = A(100 + A) and C = AB.

From this point the general formula for the Hall's separation curve is obtained as:

$$\beta = \beta_t - \frac{\beta_t - \alpha}{100}\frac{A\varepsilon}{100 + A - \varepsilon} \tag{6}$$

### *2.2. Liberation efficiency index*

Having the mathematical formula for Hall's beneficiation curve. it is possible to calculate the absolute liberation efficiency index. This value describes the percentage ratio of clean mineral substance in the final product to the total amount of this substance in the feed. This index is defined as follows:

$$E = \frac{\frac{\varepsilon_n}{100} - \frac{\alpha}{\beta_t}}{1 - \frac{\alpha}{\beta_t}} \tag{7}$$

where:

$\varepsilon_n$ – equation 8 (in the frame on the bottom of this page).

This methodology was applied to calculate the liberation efficiency for the tested coal type.

## 3. Experiment and calculations

### *3.1. Hall's separation curve*

The Hall's separation curve was used to describe the results of dense metric analysis of hard coal. type 35.2 (coking coal). The tested material was raw coal from the Upper Silesian Industrial Region. The ash contents in raw coal was about 30,1%. The coal was initially screened using coal screens with different mesh types: 20,0; 16,0; 14,0; 12,5; 10,0; 8,0; 6,3; 3,15; 1,0 mm. Then each particle size fraction was separated into density fractions in zinc chloride solution of densities: 1,3; 1,4; 1,5; 1,6; 1,7; 1,8; and 1,9 $Mg/m^3$. For each size-density fraction the ash contents was determined. The results of the experiment were presented in Table 1.

The results of dense metrical analysis presented in table 1 were depicted by the separation curve according to the formula (6). The effects of fitting theoretical separation curve to empirical points were presented in figures 1–3 for the extreme size fraction and in table 2 for each fraction. The curvilinear correlation indexes were equal to about 99%, which confirmed high compliance of the proposed model with empirical data. By applying equations (7) and (8), the absolute liberation efficiency index of mineral fraction was calculated by

$$\varepsilon_n = \left(100 + A - \frac{A\alpha}{2\beta_t}\right) - \sqrt{\left(100 + A - \frac{A\alpha}{2\beta_t}100\right)^2 - 100(100 + A)} \tag{8}$$

Table 1. Yields (γ) and ash contents ($\lambda_A$) in size-density fractions

Tabela 1. Wychody i zawartości popiołu w klaso-frakcjach

| Size of fractions [mm] | Fractions density [Mg/m³] | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | - 1,3 | | 1,3 – 1,4 | | 1,4 – 1,5 | | 1,5 – 1,6 | | 1,6 – 1,7 | | 1,7 – 1,8 | | 1,8 – 1,9 | | + 1,9 | |
| | γ [%] | $\lambda_A$ [%] | γ [%] | $\lambda_A$ [%] | γ [%] | $\lambda_A$ [%] | γ [%] | $\lambda_A$ [%] | γ [%] | $\lambda_A$ [%] | γ [%] | $\lambda_A$ [%] | γ [%] | $\lambda_A$ [%] | γ [%] | $\lambda_A$ [%] |
| 0–1,0 | 67,56 | 1,84 | 9,38 | 7,07 | 3,05 | 17,44 | 1,27 | 25,96 | 1,03 | 33,76 | 1,06 | 38,64 | 0,59 | 47,17 | 16,06 | 77,49 |
| 1,0–3,15 | 54,7 | 2,22 | 12,45 | 7,84 | 4,16 | 17,61 | 1,88 | 27,7 | 1,84 | 35,57 | 1,45 | 43,45 | 1,15 | 50,64 | 22,38 | 81,31 |
| 3,15–6,3 | 37,86 | 2,49 | 13,56 | 8,3 | 5,36 | 18,98 | 2,80 | 27,9 | 2,48 | 34,64 | 1,68 | 41,2 | 1,74 | 48,57 | 34,53 | 77,16 |
| 6,3–8,0 | 31,33 | 3,88 | 10,54 | 9,78 | 5,49 | 17,92 | 3,50 | 29,19 | 2,44 | 38,91 | 2,18 | 44,58 | 2,08 | 49,87 | 42,43 | 78,62 |
| 8,0–10,0 | 35,68 | 2,32 | 8,27 | 9,18 | 4,22 | 18,73 | 7,91 | 27,12 | 2,65 | 35,35 | 2,18 | 35,14 | 1,93 | 51,41 | 37,16 | 79,78 |
| 10,0–12,5 | 29,86 | 4,2 | 12,09 | 9,25 | 4,55 | 20,57 | 2,13 | 28,56 | 3,61 | 36,44 | 2,81 | 45,46 | 2,32 | 55,22 | 42,63 | 78,79 |
| 12,5–14,0 | 24,60 | 2,38 | 8,18 | 8,97 | 3,64 | 19,61 | 2,01 | 35,68 | 2,35 | 34,62 | 2,66 | 40,6 | 2,57 | 52,24 | 53,98 | 80,57 |
| 14,0–16,0 | 21,77 | 2,62 | 9,63 | 9,15 | 4,42 | 20,13 | 1,65 | 25,02 | 4,30 | 36,36 | 3,12 | 43,43 | 4,21 | 52,4 | 50,89 | 52,02 |
| 16,0–20,0 | 22,08 | 2,27 | 7,84 | 9,9 | 5,20 | 18,15 | 2,32 | 26,5 | 3,72 | 35,7 | 3,81 | 46,17 | 2,13 | 56,9 | 52,9 | 82,62 |

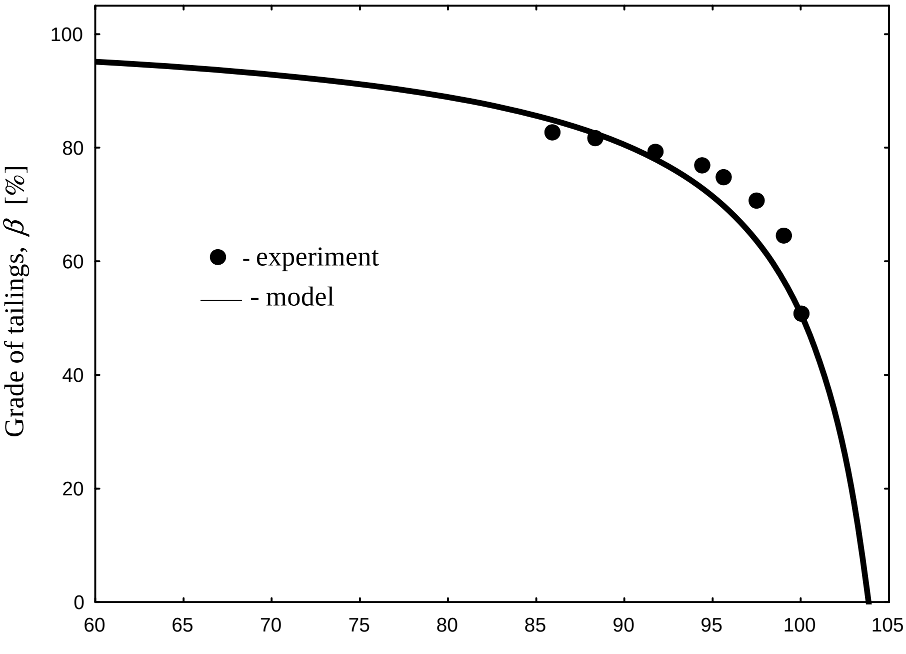


Fig. 1. Coal separation curve for fraction 16–20 mm

Rys. 1. Krzywa separacji węgla dla klasy 16–20 mm

an assumption that the ash contents in tailings $\beta_t$ for each particle size fraction was equal to 100%. The value of the absolute liberation efficiency index *E* and the equation describing the separation curve for particle size fraction were given in the figures 1–3 and table 2.

By analyzing the relation of liberation efficiency index of mineral fraction in individual particle size fractions, it can be observed that the value of this index grows as the particle size becomes smaller. However, the growth of the absolute liberation efficiency is not big because of the fact that the tested coal was quite homogenous. For the finest size fraction, 0–1 mm the index was equal to 73,7%.

## 4. Conclusions

Calculated liberation factors (*A*) and liberation efficiencies (*E*) characterize precisely liberation of mineral phase of coal compounds. In investigated coal the ash contents in wastes ($\beta_t$) being equal to 100% was evaluated what means that maximum liberation of mineral phase was assumed. However, this assumption is not always correct. It is conditioned mainly by coal type what determines its real contents and several parameters connected with physicochemical properties of coal.

Analyzing the results of the investigation. it can be noticed that the model of Hall's separation curve presented in this paper and the absolute liberation

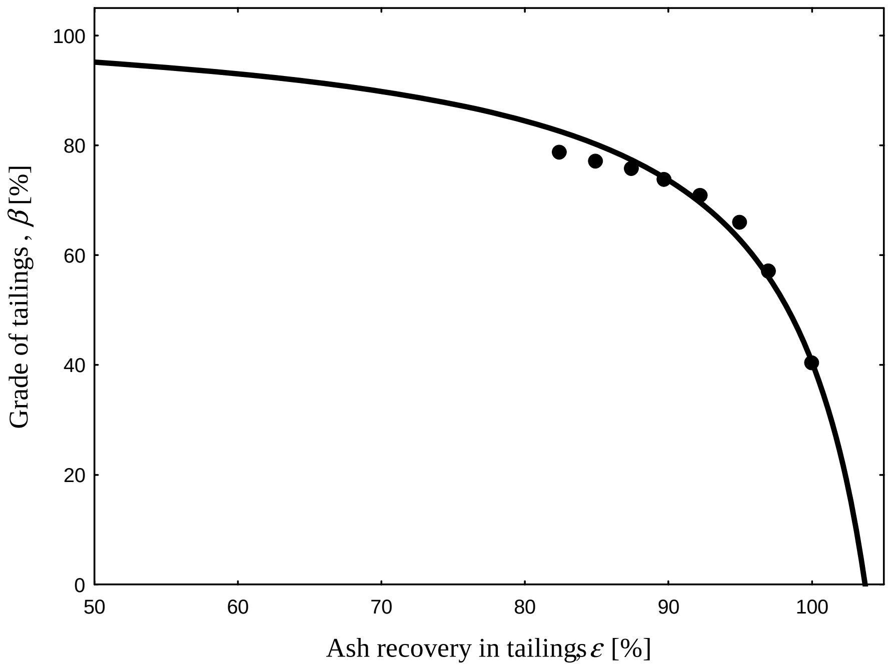


Fig. 2. Coal separation curve for fraction 6,3–8 mm

Rys. 2. Krzywa separacji węgla dla klasy 6,3–8 mm

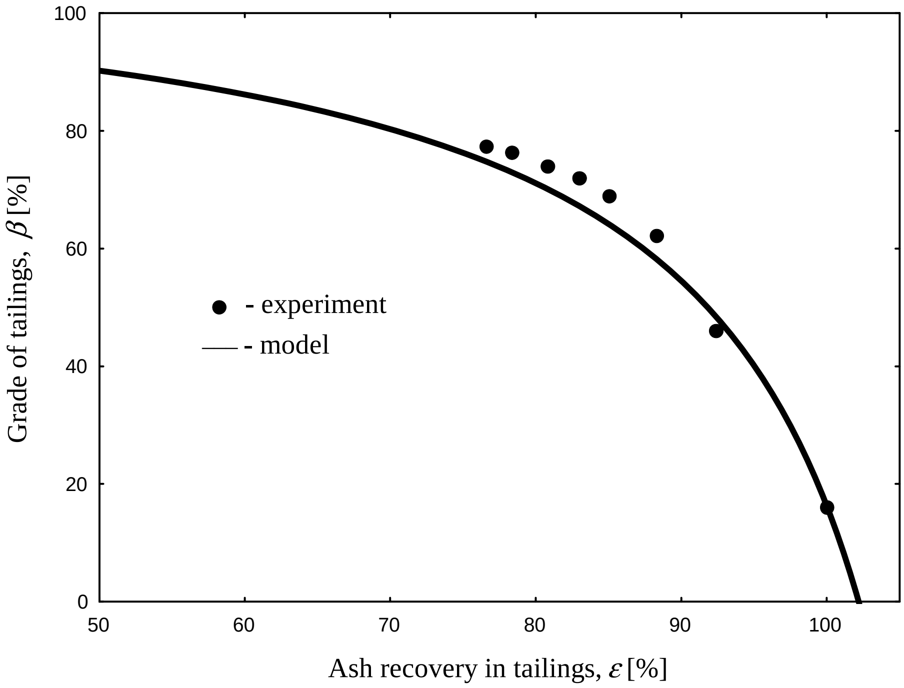


Fig. 3. Coal separation curve for fraction 0–1.0 mm

Rys. 3. Krzywa separacji węgla klasy 0–1.0 mm

efficiency index calculated on its basis describes well enough the liberation of mineral fraction from coal. An additional advantage of such description of the separation process is the fact that it can be presented by mathematical equation of hyperbola. It can provide the basis not only for the evaluation of the mineral fraction liberation, but also for determination of $\beta_t$ value because the equation contains this value.

The advantage of Hall's separation curve in comparison with other ones describing hard coals beneficiation is the fact that it is possible to evaluate mineral phase liberation efficiency and precise the value of $\beta_t$.

Table 2. Equations of Hall's separation curve liberation factor and absolute liberation efficiency index in size fractions

Tabela 2. Równania krzywej separacji Halla oraz współczynnik uwolnienia i absolutny stopień uwolnienia w klasach ziarnowych

| Size of fractions [mm] | Equation of Hall's separation curve | Liberation factor A | Absolute liberation efficiency index E [%] |
|---|---|---|---|
| 0 – 1 | $\beta = 100 - \frac{9{,}38\varepsilon}{111{,}2 - \varepsilon}$ | 11,2 | 73,7 |
| 1 – 3,15 | $\beta = 100 - \frac{8{,}23\varepsilon}{110{,}8 - \varepsilon}$ | 10,8 | 72,9 |
| 3,15 – 6,3 | $\beta = 100 - \frac{7{,}26\varepsilon}{110{,}8 - \varepsilon}$ | 10,8 | 71,1 |
| 6,3 – 8 | $\beta = 100 - \frac{5{,}76\varepsilon}{109{,}7 - \varepsilon}$ | 9,7 | 70,6 |
| 8 – 10 | $\beta = 100 - \frac{7{,}42\varepsilon}{111{,}8 - \varepsilon}$ | 11,8 | 69,4 |
| 10 – 12,5 | $\beta = 100 - \frac{6{,}4\varepsilon}{110{,}9 - \varepsilon}$ | 10,9 | 69,2 |
| 12,5 – 14 | $\beta = 100 - \frac{4{,}6\varepsilon}{109{,}1 - \varepsilon}$ | 9,1 | 69,0 |
| 14 – 16 | $\beta = 100 - \frac{4{,}15\varepsilon}{108{,}2 - \varepsilon}$ | 8,23 | 68,8 |
| 16 – 20 | $\beta = 100 - \frac{4{,}82\varepsilon}{109{,}8 - \varepsilon}$ | 9,8 | 67,8 |


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## *Ocena stopnia uwolnienia frakcji mineralnej węgla*

*Podstawowym narzędziem pozwalającym prognozować efekty wzbogacania grawitacyjnego węgli kamiennych jest analiza densymetryczna. W związku z tym otrzymane wyniki takiej analizy powinny odzwierciedlać rzeczywisty stopień uwolnienia minerałów w powiązaniu z ich właściwości fizycznymi. W celu wyliczenia uwolnienia fazy mineralnej z węgla kamiennego wykonano analizę granulometryczną i densymetryczną surowego węgla kamiennego – nadawy kierowanej do przeróbki mechanicznej. W otrzymanych klaso-frakcjach wyliczono wychody oraz oznaczono zawartość popiołu.*
*W artykule wykorzystano krzywą separacji Halla do oceny stopnia uwolnienia frakcji mineralnej z węgla kamiennego. Zaletą krzywej separacji Halla jest to, że można ją opisać w sposób matematyczny przy pomocy równania hiperboli. Stwierdzono, że ze wraz z zmniejszaniem się wielkości ziaren wzrasta stopień uwolnienia substancji mineralnej.*


*Słowa kluczowe: stopień uwolnienia, węgiel kamienny, skład ziarnowy skład densymetryczny*